DECAY LAW OF THE NUCLEAR STATES OF SCATTERING IN THE REGION OF THE NEUTRON RESONANCES

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We calculated the spectra of delay times for neutron scattering by the nuclides of ^{58,60,61}Ni in a forward direction, on the basis of the experimental data on the energy dependence of total neutron cross-sections. Calculations were carried out for the interval of incident energies in the neutron beam $700 > E_n > 600$ keV. The mean lifetimes of the corresponding intermediate states of scattering and their relative standard deviations were found. The obtained time spectra demonstrate the non-monotone decrease in the form damped oscillations. A difference in the law of decay from the smooth exponential curve gives grounds to consider that in these cases the decay occurs from the states, which are not statistically equilibrium. It was shown that the time spectra are in the qualitative agreement with the approximation, whose four parameters make simple physical sense. This approximation is based on the assumption about the retention of the periodicity of motion in the open composite nuclear system, in spite of the fluctuation of the resonance spacing and widths.

1. Introduction

It is usually assumed that neutron scattering by the nuclei in the region of resonance energy was already sufficiently well studied. Nevertheless, a study of chaotic nature of the intermediate states, which appear in this case, is still of significant interest [1]. Further development obtained studies of the statistical properties of delay times in the compound systems [2]. Progress occurs also in a study of non-statistical effects of such as doorway states [3]. The study of evolution in the time of the intermediate states of the nuclei, which are formed with scattering of the resonance neutron beams of broad energy spectrum, is the object of this work. The history of problem and some new data, connected with this work we examined into [4].

It should be noted that for the compound scattering by the nuclei of the medium atomic weight of neutrons with the energies in several hundred keV mean delay times are expected order 10^{-18} s. For the direct measurement of the time delay distributions with the scattering it is necessary to fix intensity on even the shorter time intervals. However, a sufficiently precise experimental technique in this case, generally speaking, thus far is absent. Therefore the analysis of the corresponding time spectra, obtained by calculation, can be fruitful. We saw by our task the development of the specific procedure of calculation, which to the greatest degree uses the existing data on the measurement of neutron cross sections.

2. Calculation of the time spectra of the delayed events for scattering the resonance neutrons by the nuclei of ^{58,60,61}Ni

A study of the development of nuclear reaction in the time becomes expedient, if beams of particles with the broad energy spectrum initiate it. In this case it is possible to fix the moment of collision with the accuracy $\Delta t_0 = \hbar/\Delta E$. Here ΔE is energy spread in the incident wave packet with the mean energy of its relative motion $\langle E \rangle$. Sufficiently large energy spread ΔE of the beam makes it possible to determine the time intervals $t > \Delta t_0$, through which the particles appear in the outgoing channel j at angle θ . Then measurement of the corresponding intensity will give density of probability distribution $P_{ji}(t,\theta)$ for the reaction channel $i \to j$. If we do not take into consideration the matrix nature of the amplitude of events $f_{ji}(E,\theta)$, then the corresponding time spectrum $P_{ji}(t,\theta)$ through the Fourier transform is connected with the power spectrum $S_{ji}(t,\theta)$ for the process of collision:

$$P_{ji}(t,\theta) = \frac{S_{ji}(t,\theta)}{\int_{-\infty}^{+\infty} S_{ji}(t,\theta)dt} \tag{1}$$

$$S_{ji}(t,\theta) \propto \left| \int_{E_{\min}}^{E_{\max}} W(E) f_{ji}(E,\theta) \exp(-iEt/\hbar) dE \right|^2 . \tag{2}$$

Here $(E_{\max} - E_{\min}) = \Delta E$ is determined by boundary energies of particles in the incident beam and $W(E)$ is the window function, which decreases the spectral bias. For process of elastic neutron scattering $n \to n$ considered here neglect of the matrix nature of amplitude is strictly correct for the scattering on the even-even nuclei in a forward or backward direction ($\theta = 0,\pi$). For the scattering angles $\theta = 0,\pi$, where there is no polarization, this acceptably also for

the targets with the spin not equal to zero. Similarly for the low energies it is possible not to consider the small contribution of spin-orbit interaction for the component of scattering amplitude $f_{nn}^{c}(E,\theta)$, connected with the formation and the subsequent decay of composite system.

Specifically, such situations are examined subsequently, where the discussion will deal with the time delay for forward scattering of resonance neutrons by the nuclides ^{58,60,61}Ni.

It is obvious that relationships (1) – (2) will be useful for finding the time spectrum only in the case, when scattering amplitude is known. We intend to calculate the time distributions of delays, connected with the decay of the intermediate states of the nuclei, which are formed in the process of scattering. Therefore we should know scattering amplitude with the formation of composite system, after excluding the contribution of direct instantaneous component:

$$f_{nn}^{c}(E,\theta) = f_{nn}(E,\theta) - \left\langle f_{nn}(E,\theta) \right\rangle_{\Delta E}, \tag{3}$$

where

$$\left\langle f_{nn}(E,\theta) \right\rangle_{\Delta E} = \frac{\int_{E_{\min}}^{E_{\max}} f_{nn}(E,\theta)dE}{(E_{\max} - E_{\min})}. \tag{4}$$

Specifically, the amplitude $f_{nn}^{c}(E,\theta)$ must figure in formula (2) for calculating the time spectrum of scattering with the delay. In accordance with what has been said earlier we will limit to the examination only of case of forward scattering $\theta = 0$. However, the expected weak anisotropy for the scattering with the delay makes it possible to hope for the applicability of basic regularities also for other scattering angles.

Thus, now task is reduced to finding of the corresponding amplitude. For this purpose we used the fact that the optical theorem connects total neutron cross-section with the imaginary part of the amplitude of the elastic neutron scattering of the same energy in a forward direction.

Let us assume that the real and imaginary parts of amplitude (3) of the process delayed in the time behave similarly to the statistically independent random values with the identical distributions. Then it seems that the density of the time delays distribution $P_{nn}^{c}(t,0^{\circ})$, calculated through the amplitude $f_{nn}^{c}(E,0^{\circ})$, and the distribution density $P_{nn}^{\mathrm{Im}\, f^{c}}(t,0^{\circ})$, calculated only through its imaginary part $\mathrm{Im}\, f_{nn}^{c}(E,0^{\circ})$, will be also identical. The validity of this assumption was confirmed by numerical calculations with the model amplitude of resonance scattering. Model [4] takes into consideration the statistics of the resonance spacing and the statistics of widths. The parameters of model are attached to the experimental values averaged on the energy. Fig. 1 shows the comparison of two model time distributions, calculated with the use of composite amplitude (3) and with the use only of its imaginary part for forward scattering of neutron beam with the energies in the interval $700 > E > 600$ keV on the target of ^{60}Ni. Identity of both distributions is obvious.

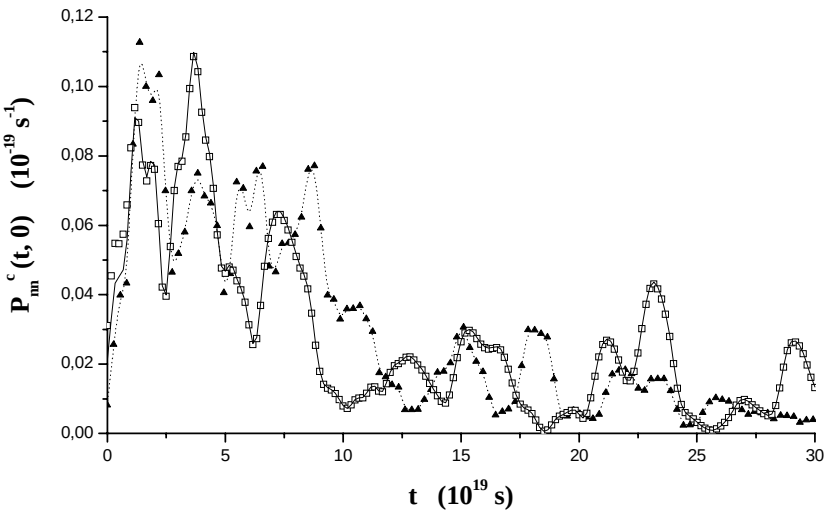

Fig. 1. The time delay distributions for the zero-angle scattering of neutron beam with the energies $700 > E > 600$ keV on the target of ^{60}Ni: calculated with the use of model [4] composite amplitude (3) (open squares) and calculated with the use only of its imaginary part (solid line). The triangles show the same time delay distribution, calculated on the basis of the measured total neutron cross-sections [5]; the dotted line connects data points and serves as a guide for the eye.

The basic calculations of the spectra of delay times were also fulfilled for forward scattering of neutron beams with the energies in the interval $700 > E > 600$ keV on the nuclides of ^{58,60,61}Ni. Necessary for the calculation data of high resolution in the energy dependence of total neutron cross-sections $\sigma_t(E)$ were undertaken from the works [5, 6]. The

step of the corresponding excitation functions made it possible to obtain as a result the procedure of fast Fourier transform the time spectra in the range of the delays $(0-110)\cdot 10^{-19}$ s with the step $\delta t_0 \cong 0.27\cdot 10^{-19}$ s. The results were obtained with Welch window and smoothed in the time intervals $\delta t = 1.35\cdot 10^{-19}$ s for decreasing the spread of points, inherent in periodogram (2). Fig. 2, *a*, *b* and, *c* demonstrate respectively for ^{58}Ni, ^{60}Ni and ^{61}Ni the initial sections of these spectra of delay times, where the intensity is sufficiently great. As can be seen, their nonmonotonic nature is the basic special feature of the time spectra shown in the Fig. 2. They are the sequence of the almost periodic peak structures with the smoothly being decreased amplitudes. Returning to Fig. 1, we can also note the qualitative agreement of the behaviour of time delay distributions for neutron scattering on the nuclei of ^{60}Ni, calculated on the basis of model [4] and the calculated on basis experimental total neutron cross-sections of this isotope [5]. Both calculations lead in this case also to the close values of the relative standard deviation of delay time $\delta t_c / \langle t_c \rangle \approx 1.2$ and mean delay time ($\langle t_c \rangle \approx 16\cdot 10^{-19} s$ for the model [4] and $\langle t_c \rangle \approx 14\cdot 10^{-19} s$ for the data of work [5]).

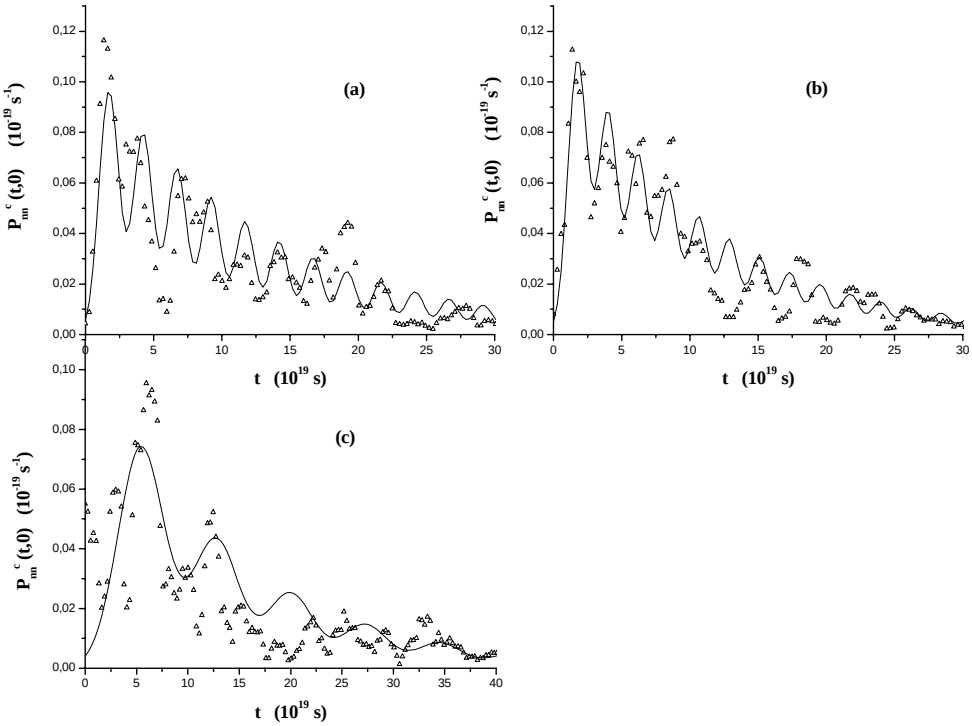

Fig. 2. Time spectra (open triangles) of the delayed events, calculated on the basis of data [5, 6] for forward scattering of neutron beam with the energies $700 > E > 600$ keV on the target of ^{58}Ni (*a*), ^{60}Ni (*b*) and ^{61}Ni (*c*). Solid lines show their approximation by expression (5).

3. Interpretation of the decay of scattering states

The represented time distributions of delays relate the decay of the intermediate states of the scattering, which are formed, when target nuclei absorb the beams of resonance neutrons with the broad energy spectrum. Following [7] we will consider that the periodic repetition of identical configurations is the characteristic property of intranuclear motion. Let us in particular determine as the repetition period T of initial configuration the minimum time interval between the moments, when the incoming particle only begins to penetrate the target nucleus and when it again appears on the surface of nucleus. The exit of this particle into the space of open channels here becomes possible. However, the degree of the penetrability of nuclear surface limits the escape probability of particle after one period by value $W_e < 1$. Then the probability of emission after k periods will be $W(k) = (1-W_e)^{k-1} W_e$. The decay of composite system can occur at any stage in the interval of values $\infty > k \geq 1$.

Therefore as the approximation of the density of the time delays distribution we will use the following expression:

$$P^c(t) = \frac{2\sum_{k=1}^{\infty}\{\alpha \exp[-\alpha^2 (t-kT+L)^2]\} Q^{k-1}}{\sqrt{\pi}\sum_{k=1}^{\infty}[1+erf(\alpha kT - \alpha L)] Q^{k-1}} \qquad \text{for } t \geq 0,$$

$$P^c(t) = 0 \qquad \text{for } t < 0. \qquad (5)$$

Here denominator and coefficient of 2 in the numerator appear as a result normalization $\int_0^\infty P^c(t)dt = 1$. Gauss form in the curly braces approaches the delta-function of Dirac, when the parameter $\alpha \to \infty$. Then $P^c(t)$ is regular sequence δ-pulses. For finite value of the parameter α the distribution $P^c(t)$ is the sequence of the pulses of Gaussian form with the half-width $\Delta T_{1/2} = 2\sqrt{\ln 2}/\alpha$. Spacing between pulses is equal to period T, and their amplitudes fall with the factor of damping $Q = (1 - W_e)$. Of this type time power spectrum should be expected, when is examined the group of resonances by the identical widths Γ_0 and spacing D_0 in energy range of the excitation $\Delta U >> D_0, \Gamma_0$ [8, 9.]. Then $T_0 = 2\pi\hbar/D_0$ is cycle time Poincare for the system of discrete equidistant levels and $Q_0 = \exp(-2\pi\Gamma_0/D_0)$. Actually emission pulses cannot have a half-width $\Delta T_{1/2} = 2\sqrt{\ln 2}/\alpha$ less than caused by energy spread ΔE in the beam. To the broadening of emission pulses and to certain finite value of the parameter α will bring also the statistics of resonance widths and spacing. In turn effective period of intranuclear motion T can differ from T_0 as the factor of the damping Q from the factor Q_0, even if in the scattering participated only one wave with the assigned spin J and the parity π. Finally the value of the parameter of the initial phase of the emission L is limited by the interval $0 \div T$.

In Fig. 2, *a*, *b* and *c* solid line showed the results of approximation by formula (5) of the obtained spectra of delay times for nuclei $^{58,\ 60,61}$Ni. The Table gives the parameters (α, T, Q, L) evaluated by the method of least squares. The corresponding coefficients of the determination R^2 are there given together with the reduced values of chi-square χ^2/NDF, where NDF is the number of degrees of freedom. Furthermore, the Table gives also the estimations of the mean lifetimes of intermediate states for the forward scattering $\langle t_c \rangle$ and their relative standard deviations $\delta t_c / \langle t_c \rangle$. They were obtained by integration the time spectra, which gives the possibility to determine moments of delay times $\langle t_c^q \rangle = \int_{-\infty}^{\infty} t^q P_{nn}^c(t, 0^\circ) dt$ order $q = 1, 2 \ldots m$.

The results of four parameters least squares fitting of the time delay spectra $P_{nn}^c(t, 0^\circ)$ with formula (5) together with respective mean time delays and their relative standard deviations are listed

$^A X$	$\alpha \cdot 10^{-19} s^{-1}$	$T \cdot 10^{19} s$	Q	$L \cdot 10^{19} s$	R^2	χ^2/NDF	$\langle t_c \rangle \cdot 10^{19} s$	$\delta t_c / \langle t_c \rangle$
^{58}Ni	0.97	2.49	0.82	0.77	0.85	$4.0 \cdot 10^{-5}$	15.4	1.10
^{60}Ni	1.00	2.22	0.81	0.46	0.92	$2.1 \cdot 10^{-5}$	13.5	1.23
^{61}Ni	0.31	7.26	0.58	1.81	0.74	$6.1 \cdot 10^{-5}$	19.9	1.14

5. Final remarks

We used Fourier analysis for investigating the evolution in the time of the intermediate nuclear states, which are formed with the resonance scattering of the neutron beams of broad energy spectrum. It was shown that for this purpose can be used the experimental data on the energy dependence of total neutron cross-sections, obtained with the high resolution. The calculations of the time delay spectra were fulfilled for forward scattering of neutron beam with the energies in the interval of 600-700 keV on the target nuclei of ^{58}Ni, 60Ni and ^{61}Ni. The integration of time spectra made it possible to find the mean lifetimes of intermediate states for the forward scattering $\langle t_c \rangle$ and their relative standard deviations $\delta t_c / \langle t_c \rangle$. The presence of the almost periodic sequence of the peaks of the decreasing amplitude is the characteristic feature of the obtained time delay distributions $P_{nn}^c(t, 0^\circ)$. The greatest deviations from the monotonic decay curve occur for delays $t \le \langle t_c \rangle$. The observed nature of decay can be understood on the basis of the idea about the fact that the oscillations are the consequence of the periodicity of motion in the composite system. It can be supported, in spite of the openness of system and fluctuation of resonance widths and spacing. On the basis of such ideas, we obtained the expression, which approximates the density of the time delay distribution. This expression reproduces rather well the obtained time spectra $P_{nn}^c(t, 0^\circ)$. Better agreement is achieved for the even-even isotopes of nickel. Parameters of least – square fit (see Table) lead to the values of the relative half-widths of the peaks of emission $\gamma = \Delta T_{1/2}/T = 2\sqrt{\ln 2}/\alpha T$, which are equal to 0.69, 0.75 and 0.73 respectively for ^{58}Ni, ^{60}Ni and ^{61}Ni. While, the analysis of expression (5) shows that for the values $\gamma \ge 1.2$ the decay mode changes from the oscillating to the aperiodic damping. In this case the decay curves (5) for weakly overlapped resonances become exponential, which would make it possible to speak about the decay of the compound nuclei from the states of statistical equilibrium. Smallness of the observed values $\gamma \approx 0.7 - 0.75$ is the sign of the fact that decaying states of scattering considered here are not in a strict sense equilibrium.

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