UDC 539.17

DIFFUSENESS OF NUCLEUS-NUCLEUS POTENTIAL AND DIFFUSENESS OF DENSITY DISTRIBUTION IN NUCLEI

V. Yu. Denisov, V. A. Nesterov

Institute for Nuclear Research, National Academy of Sciences of Ukraine, Kyiv, Ukraine

The interaction potentials between nuclei evaluated with the help of the Skyrme force in the frameworks of the extended Thomas - Fermi approximation and the Hartree – Fock - BCS theory are studied in detail. The increasing of the value of diffuseness parameter of the density distribution in interacting nuclei leads to the reduction of the barrier height between nuclei, the enlargement of the capture-well depth and the growing of the fusion cross-section. It is shown that the diffuseness parameter of nuclear part of the potential at large distances between nuclei evaluated with the help of the Skyrme force exceeds the diffuseness parameter of the nucleon density in interaction nuclei approximately in 1,5 times. The realistic values of diffuseness parameter of nuclear interaction between medium and heavy nuclei lie in the range $a \approx 0{,}75$ - 0,90 fm.

1. Introduction

In order to calculate the various characteristics of nuclear reactions, it is necessary to know the potential energy of interaction between nuclei [1 - 3]. Therefore, both the magnitude and the radial dependence of the interaction potential between nuclei at small internuclear distances are vitally important for the description of the reaction cross-section in the framework of any model.

The interaction energy of colliding nuclei is caused by both the Coulomb interaction between protons and the nuclear interaction between nucleons [1 - 3]. The Coulomb interaction between protons in nuclei is described rather well, which cannot be said about the nuclear part of nucleus-nucleus interaction. A rather large number of various approximations for the nucleus-nucleus interaction has been proposed at present [1 - 8], but they bring about different barrier heights of the nucleus-nucleus reaction [8, 9]. The barrier height depends on the ratio between the Coulomb repulsive and nuclear attractive potentials, which act at small distances between the surfaces of interacting nuclei.

In order to determine the amplitude of the nuclear interaction between nucleons which belong to different nuclei, it is desirable to use the most exact methods that have been developed for the detailed description of various characteristics of the ground and excited nuclear states [10 - 17]. Using these methods, one can calculate the energy of interaction between nuclei with high accuracy. In this work, we use both the semiclassical and semimicroscopical approaches in order to determine the interaction potential between nuclei. In the framework of the semiclassical approach, the distributions of the nucleon density of interacting nuclei and the potential energy of their interaction are calculated in the extended Thomas-Fermi (ETF) approximation with the Skyrme forces. In the framework of the semimicroscopical approach, the distributions of the nucleon density of interacting nuclei are evaluated in the Hartree – Fock - BCS model, while the potential energy of nucleus-nucleus interaction is obtained in the extended Thomas-Fermi approximation with the same Skyrme forces.

The reactions of subbarrier fusion [1 - 3, 18 - 26] are important from the viewpoint of the definition of the nuclear interaction potential, because these reactions are connected with the interaction amplitude and the potential behavior at distances close to the nucleus-nucleus touching distance. There are a lot of various models for the description of subbarrier fusion reactions [1 - 3, 18 - 26]. In order to describe the value of the fusion cross-section adequately, the fitting of the nucleus-nucleus interaction parameters is often carried out. For example, the coupled-channel analysis of the data concerning the subbarrier fusion of nuclei, fulfilled in works [22 - 26], gives a rather high value of the diffuseness ($a \sim 0{,}8$ - 1,5 fm) of the nuclear part of the nucleus-nucleus potential parametrized in the form of the Woods-Saxon potential. The authors of many other works [1 - 3, 7, 18 - 21] used lower values for the diffuseness of the nuclear part of the interaction potential between nuclei ($a \approx 0{,}6$ - 0,7 fm) to describe various nuclear reactions. Therefore, the comprehensive study of the diffuseness magnitude for the nuclear part of the interaction potential between nuclei in the framework of various models is very interesting. It is also useful to determine the realistic values for the diffuseness of the nuclear part of the interaction potential between nuclei parametrized in the form of the Woods-Saxon potential.

Our approach for evaluation of the interaction potential between nuclei is presented in the section 2. Dependences of both the nucleus-nucleus potential and the fusion cross-section on the diffuseness of the nucleon densities in both nuclei are discussed in sections 3 and 4 respectively. Conclusion is given in section 4.

2. Interaction potential between nuclei

In the framework of the "frozen density" approximation, let us define the potential energy of interaction *V(R)* between two nuclei positioned at a distance *R* between their mass centers as the difference of the binding energies $E_{12}(R)$ and $E_1 + E_2$ of the system composed of two nuclei separated by a finite *(R)* or an infinite interval, respectively [8, 9]

$$V(R) = E_{12}(R) - (E_1 + E_2)\,. \tag{1}$$

The corresponding binding energies of the nuclear system and nuclei 1 and 2 can be found easily making use of the semiclassical expression for the energy density functional if one knows the distribution of nucleon density in the nuclei

$$E_{12}(R) = \int \varepsilon\left[\rho_{1p}(\overline{r}) + \rho_{2p}(\overline{r},R),\ \ \rho_{1n}(\overline{r}) + \rho_{2n}(\overline{r},R)\right] d\overline{r}\ , \tag{2}$$

$$E_1 = \int \varepsilon\left[\rho_{1p}(\overline{r}),\ \ \rho_{1n}(\overline{r})\right] d\overline{r}\ , \tag{3}$$

$$E_2 = \int \varepsilon\left[\rho_{2p}(\overline{r}),\ \ \rho_{2n}(\overline{r})\right] d\overline{r}\ . \tag{4}$$

Energy density functional we wrote in a form

$$\varepsilon(\rho_n, \rho_p) = \tau + \varepsilon_{pot} + \varepsilon_{coul}\ , \tag{5}$$

where τ, ε_{pot} and ε_{coul} are the densities of kinetic, potential and Coulomb energies respectively. It is evident that, to determine the potential energy of interaction, we have to know the distribution of nucleon density and the energy density functional. These issues were considered in detail in Refs. [8, 9, 11 - 14]. The nucleus-nucleus potential at finite distances between the surfaces of nuclei is caused by the interaction of nucleons in the range of "overlapping tails" of the nucleon density distributions. Therefore, taking the gradient terms in the kinetic energy density into account is very important for the calculation of the potential amplitudes to be accurate.

3. Diffuseness of the density distribution and the properties of the nucleus-nucleus interaction potential

In order to study the influence of the density diffuseness of colliding nuclei, which are in the ground state, on the nucleus-nucleus potential, let us parametrize the nucleon densities of nuclei in the ground states by the expression

$$\rho_{n(p)}(r) = \rho_{0n(p)} / \left\{\ 1 + \exp[(r - R_{n(p)})/d\right\}. \tag{6}$$

Such a parametrization of the radial distribution of nucleon density has often been used in nuclear physics [27]. The parameters of this distribution $\rho_{0n(p)}$ and $R_{n(p)}$ were found by the direct variational method at a fixed value of the diffuseness *d*. By varying $\rho_{0n(p)}$ and $R_{n(p)}$, we minimized the nucleus binding energy calculated taking into account the gradient corrections to the kinetic energy functional and Skyrme forces SkP. The diffuseness *d* for the neutron and proton densities was varied within the interval from 0,5 to 0,8 fm with a step of 0,05 fm.

The radial distributions of the neutron and proton densities of ^{16}O and ^{208}Pb nuclei in their ground states with such values of the diffuseness *d* are plotted in Fig. 1. These densities were used to calculate the interaction potentials between ^{16}O and ^{208}Pb nuclei, which are also displayed in this figure. While calculating the potentials, the ETF method with Skyrme forces SkP was used. For a comparison, the results of calculations of the potential between those nuclei with nucleon densities of HF-BCS approximation is shown (see Fig. 1). At large distances between nuclei, the potential calculated with nucleon densities of HF-BCS approximation is close to the potential which uses the density parametrization (1) with the diffuseness *d* = 0,55 fm. However, at shorter distances, it is close to the potential calculated with the diffuseness of about 0,6 fm (see Fig. 1).

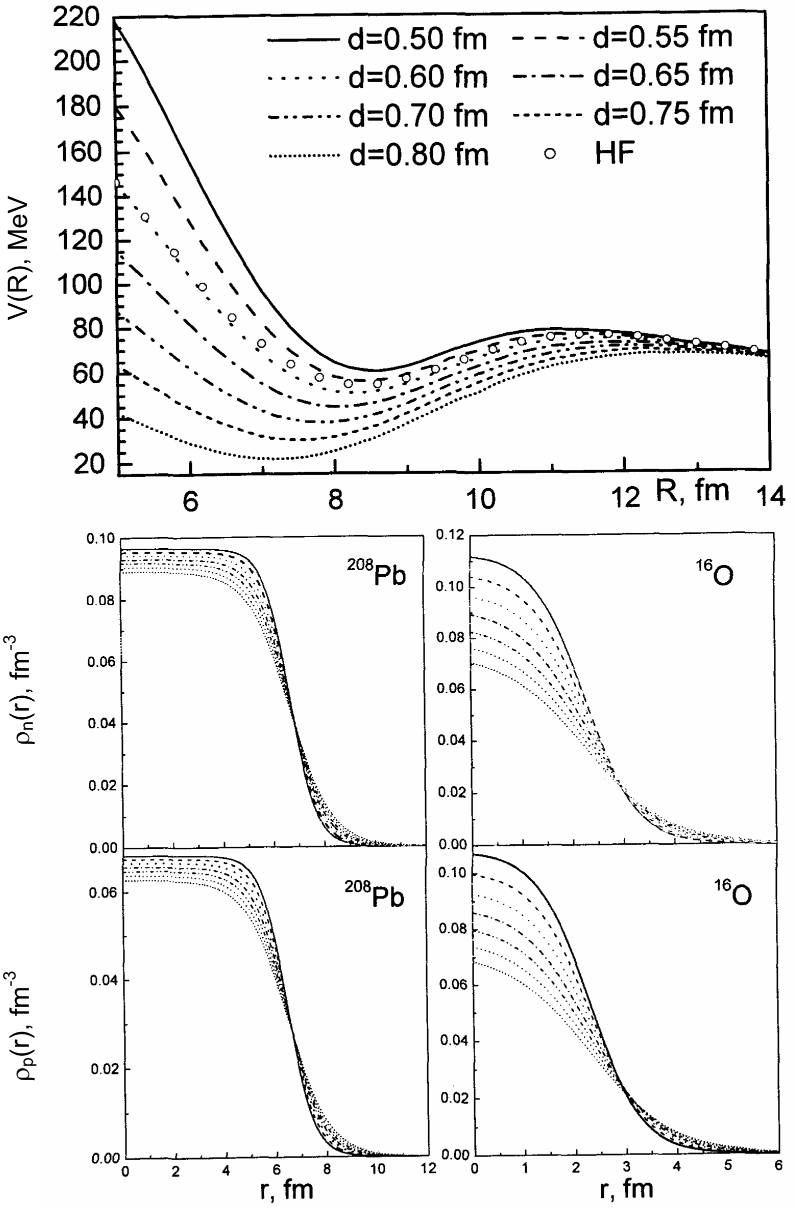

Fig. 1. Interaction potentials *V(R)* for ^{16}O and ^{208}Pb nuclei for various values of the diffuseness *d* of the ground state densities calculated in the semiclassical approximation. For comparison, the interaction potential between nuclei found with nucleon densities of HF-BCS approximation is shown, as well as the proton and neutron densities in the ground state for various *d*.

As the diffuseness of the density distribution grows, the potential capture well shifts towards longer distances between colliding nuclei, its depth increases, see the Fig. 1. Interaction potentials *V(R)* for ^{16}O and ^{208}Pb nuclei for various values of the diffuseness *d* of the ground state densities calculated in the semiclassical approximation. The reason is that the nucleon densities become more extended as the density distribution diffuseness grows, so that the nuclear interaction at large distances between nuclei increases, and the barrier height decreases. As the density distribution diffuseness grows, the nuclear densities come into strong overlapping at shorter distances between nuclei, so that the nuclear repulsion between the nuclei, caused by the compression of the nuclear matter, decreases. This results in the widening and deepening of the capture well.

Fig. 2 shows the dependences of the potential barrier height and the bottom energy of the capture well on the diffuseness *d* of the ground state density. The potential barrier height decreases almost linearly with the growth of the diffuseness *d,* whereas the bottom energy of the capture well decreases much more quickly. The depth of the well, i.e. the difference between the barrier height and the well bottom, increases with the growth of *d.*

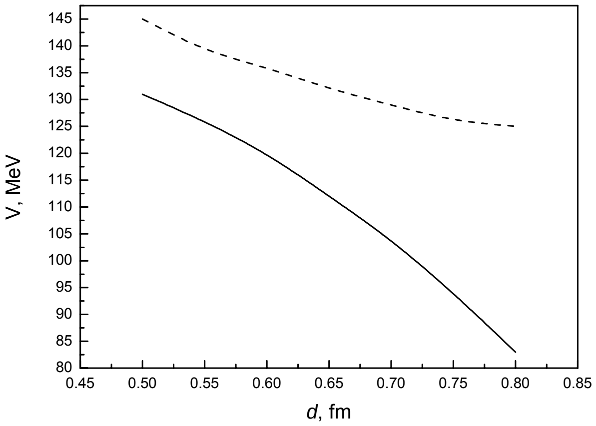

Fig. 2. The dependences of the potential barrier height and the bottom energy of the capture well on the diffuseness *d* of the ground state density for ^{16}O and ^{208}Pb nuclei.

The HF-BCS approximation describes well the experimental radial distributions of the nucleon density; therefore, the ETF potential evaluated with the help of HF-BCS nucleon densities is the most realistic. Due to this the potentials calculated making use of the density parametrization (6) with the diffuseness $d \approx 0{,}55 \div \div 0{,}6$ fm are close to the realistic one at various distances between nuclei.

4. Diffuseness of the potential and the nuclear fusion cross-section

The characteristics of nuclear reactions are often calculated making use of the Woods-Saxon parametrization of the nuclear part of the interaction potential between nuclei [1 - 3, 7, 18 - 20, 22 - 24]

$$V(R)=-V_0/\{1+exp[(R-R_{\mathrm{pot}})/a]\}. \tag{7}$$

Therefore, it is necessary to determine at what value of the parameter *a* of the Woods - Saxon potential is close to the realistic potential, found in the HF-BCS approximation. For this purpose, we determined first the parameters V_0, R_{pot}, and *a* in Eq. (7) by fitting the potential which had been calculated in the ETF approximation with the value $d = 0{,}55$ fm for the diffuseness of the nucleon density distribution, in the range of distances *R* larger than the sum of nucleus radii R_t. Afterwards, we fixed the obtained value of $R_{pot} \approx \approx 9{,}7$ fm and used it when fitting the potential which had been found in the ETF approximation for other values of the nucleon density distribution diffuseness. The dependences of V_0 and *a* on *d* are given in the Fig. 3.

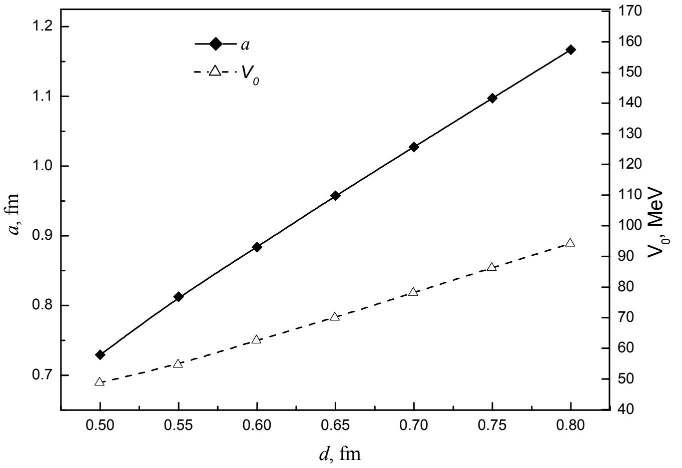

Fig. 3. Dependences of the diffuseness *a* and the depths V_0 of the Woods-Saxon potential on the diffuseness *d* of the nucleon density in the ground state for the system ^{16}O and ^{208}Pb.

The diffuseness *a* and the depth V_0 of the Woods-Saxon potential grow linearly as the nucleon density diffuseness *d* increases (see Fig. 3). The diffuseness of the nuclear part of the potential between nuclei, calculated by the ETF method and using Skyrme forces, is approximately 1,5 times larger at large distances than the diffuseness of the nucleon distribution in interacting nuclei.

As was pointed out above, the potential, which had been calculated, using the nucleon density in form (6), for the density distribution d = 0,55 fm, corresponds at large distances to the potential determined for the HF-BCS distributions of nucleon density. This value of the density diffuseness corresponds to the diffuseness of the Woods-Saxon potential $a \approx 0,82$ fm (see Fig. 3). The Woods-Saxon potential agrees well at large distances with the nuclear part of the potential calculated in the ETF approximation for the density distribution diffuseness d = 0,55 fm. However, in the internal nucleus region, the Woods-Saxon potential tends to V_0. Therefore, they appreciably differ there.

The obtained value of the Woods-Saxon potential diffuseness, $a \approx 0,82$ fm, is in agreement with those proposed earlier. For example, the very close value of the diffuseness of the nucleus-nucleus potential at large distances, a = 0,788 fm, was found in work [8]. A somewhat smaller value of the diffuseness, a = 0,7176 fm, was obtained in Ref. [5]. In Ref. [7], by analyzing the elastic scattering of nuclei, the value a = 0,657 fm, which is very close to a = 0,65 fm proposed by Bass in 1980 [1], was found. In Ref. [28], where various heavy-ion fusion reactions were studied, three values of the diffuseness have been proposed for light (a = 0,481 fm), medium (a = 0,675 fm), and heavy (a = 0,895 fm) systems of interacting nuclei. The analysis of data concerning the subbarrier fusion of various nuclei, carried out in works [22 - 26], led to rather large values of the diffuseness $a \approx 0,8 \div 1,5$ fm; and, while studying the subbarrier fusion of ^{16}O and ^{208}Pb nuclei, the value a = 1,005 fm was obtained [26]. The values of the diffuseness calculated in the case of the interaction between medium and heavy nuclei, which substantially deviate from $a \approx 0,82$ fm, do not consist with the realistic distributions of the nucleon density in nuclei and nucleon-nucleon forces.

In order to investigate the influence of the potential diffuseness on the cross-section of near-barrier fusion, we fulfilled calculations for the reaction between ^{16}O and ^{208}Pb nuclei making use of the CCFULL code [20]. This code calculates the cross-sections of nuclear fusion taking into account the coupling between channels with low-lying multipole surface-vibration excitations in both nuclei. In so doing, the nuclear part of the interaction potential between nuclei is parametrized in the form of the Woods-Saxon potential (7). The code takes into account the nonlinear effects of coupling with many-phonon multipole excitations of the surface. The parameters of the 2^+ and 3^- excitations, which are necessary for calculating the cross-sections with the help of the CCFULL code, were taken from the corresponding compilations of experimental data [29, 30]. The parameters for the nuclear interaction potential were the same as in the Fig. 3.

The results of fusion cross-section calculations for the reaction between ^{16}O and ^{208}Pb and the experimental data [26] are compared in Fig. 4. It is evident that the potentials calculated with small values of the density diffuseness result in the strongly underestimated cross-sections of fusion within the whole energy range. As the diffuseness of the nucleon density distribution d grows, the fusion cross-section increases, which is connected with the lowering of the fusion barrier height. However, the slopes of the cross-section vs energy curves remain practically constant at subbarrier energies for various d. We emphasize that we did not intend to describe the fusion cross-section, because our main purpose was to reveal the connection between the diffuseness of the density distribution, the diffuseness of the Woods - Saxon potential, and the fusion cross-section.

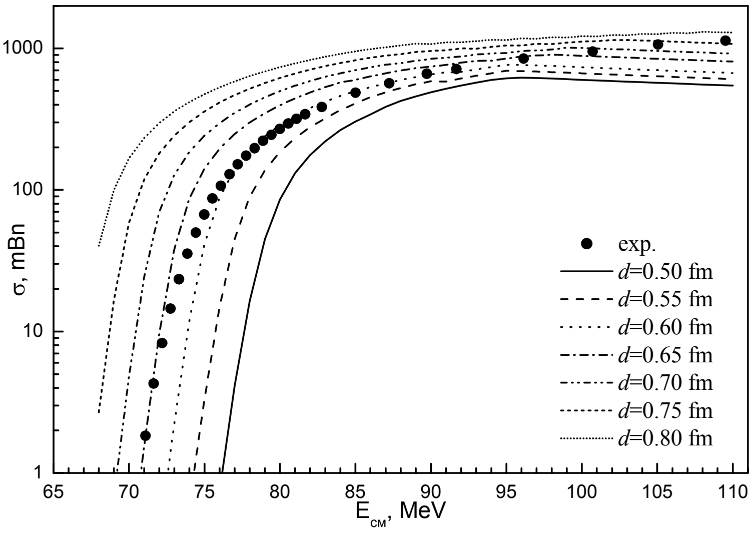

Fig. 4. Energy dependences of the cross-section of the nuclear fusion reaction ^{16}O and ^{208}Pb for various diffusenesses d.

5. Conclusions

We have calculated the interaction potentials between nuclei in the framework of the semimicroscopical HF-BCS theory and the semiclassical ETF approximation, by making various assumptions concerning the nucleon density distribution in the ground state of nuclei. The obtained potentials were calculated in the "frozen density" approximation which is valid for the near-barrier and higher energies of collisions. The barrier heights agree well with various approximations that were proposed earlier for the nucleus-nucleus interaction. The variation of the isotopic composition of interacting nuclei has been demonstrated to affect the height and the thickness of the fusion barrier substantially.

The diffuseness of the nucleon density distribution in nuclei is rigidly bound with the diffuseness of the nuclear interaction potential. The diffuseness of the potential is approximately 1,5 times higher than that of the density distribution and is close to $a \approx 0{,}82$ fm. The values of the diffusenesses of the charge distribution and, owing to the isotopic symmetry, the neutron density distribution in medium and heavy spherical nuclei are practically constant and close to $d = 0{,}55$ fm (see Table 6,3 in Ref. [27]). Therefore, the diffuseness of the potential in the case of the interaction between either medium or heavy spherical nuclei can accept neither very low nor very high values.

REFERENCES

1. *Bass R.* Nuclear reactions with heavy ions. - Berlin: Springer-Verlag, 1980. - 410 p.
2. *Satchler G.R.* Direct nuclear reactions. - Oxford: Clarendon Press, 1983. - 833 p.
3. *Frobrich P., Lipperheide R.* Theory of nuclear reactions. - Oxford: Clarendon Press -1996, 467p.
4. Blocki J., Randrup J., Swiatecki W. J., Tsang C. F. Proximity forces // Ann. Phys. - 1977. - Vol. 105, No. 2. - P. 427 - 462.
5. *Myers W. D., Swiatecki W. J.* Nucleus-nucleus proximity potential and superheavy nuclei // Phys. Rev. C - 2000. - Vol. 62 - P. 044610.
6. *Krappe H.J., Nix J.R., Sierk A.J.* Unified nuclear potential for heavy-ion elastic scattering, fusion, fission, and ground-state masses and deformations // Phys. Rev. C - 1979. - Vol. 20. - P. 992 - 998.
7. *Winther A.* Dissipation, polarization and fluctuation in grazing heavy-ion collisions and the boundary to the chaotic regime // Nucl. Phys. A - 1995. - Vol. 594, No. 2. - P. 203 - 245.
8. *Denisov V.Yu.* Interaction potential between heavy ions // Phys. Lett. B - 2002. - Vol. 526, No. 3 - 4. - P. 315 - 321.
9. *Denisov V.Yu., Norenberg W.* Entrance channel potentials in the synthesis of the heaviest nuclei // Eur. Phys. J. - 2002. -Vol. A15, No. 3. - P. 375 - 388.
10. *Ring P., Schuck P.* The Nuclear Many-Body Problem. - New York: Springer-Verlag, 1980. - 704 p.
11. *Brack M., Guet C., Hakanson H.-B.* Selfconsistent semiclassical description of average nuclear properties - a link between microscopic and macroscopic models // Phys. Rep. -1985. - Vol. 123, No. 5. - P. 275 - 364.
12. *Brack M., Bhaduri R. K.* Semiclassical physics -Addison Wesley Publ. Co, 1997. - 484 p.
13. *Денисов В. Ю., Нестеров В. А.* Энергии связи и распределения плотности атомных ядер в рамках нелокального модифицированного метода Томаса - Ферми // Яд. физ. - 2002. - Т. 65, № 5. - С. 814 - 823.
14. *Нестеров В. О.* Застосування квазікласичного підходу до аналізу легких атомних ядер з великим надлишком нейтронів // УФЖ - 2004. - Т. 49, № 3. - С. 225 - 228.
15. *Vatherin D., Brink D.* Hartree-Fock Calculations with Skyrme's Interaction. I. Spherical Nuclei // Phys. Rev. C - 1972. - Vol. 5. - P. 626 - 638.
16. *Dobaczewski J., Flocard H., Treiner J.* Hartree-Fock-Bogolyubov description of nuclei near the neutron-drip line // Nucl. Phys. A - 1984. - Vol. 422, No. 1. - P.103 - 139.
17. *Negele J. W.* The mean-field theory of nuclear structure and dynamics // Rev. Mod. Phys. - 1982. - Vol. 54. - P. 913 - 1015.
18. *Dasgupta M., Hinde D. J., Rowley N.* et al. Measuring Barriers to Fusion // Annu. Rev. Nucl. Part. Sci. - 1998. - Vol. 48. - P. 401 - 461.
19. *Balantekin A. B., Takigawa N.* Quantum tunneling in nuclear fusion // Rev. Mod. Phys. - 1998. - Vol. 70. - P. 77 - 100.
20. *Hagino K., Rowley N., Kruppa A. T.* A program for coupled-channel calculations with all order couplings for heavy-ion fusion reactions // Comp. Phys. Comm. - 1999. - Vol. 123, No. 1 - 3. - P. 143 - 152.
21. *Denisov V. Yu.* Subbarrier fusion of heavy ions and subbarrier few-nucleon transfers: Fusion of nuclei far from the beta-stability line // Phys. At. Nucl. - 1999. - Vol.62, No. 8. - P. 1349 - 1362; Subbarrier heavy ion fusion enhanced by nucleon transfer // Eur. Phys. J. - 2000. - Vol. A7, No. 1. - P. 87 - 99.
22. *Hagino K., Rowley N., Dasgupta M.* Fusion cross sections at deep sub-barrier energies // Phys. Rev. C - 2003. - Vol. 67. - P. 054603.
23. *Hinde D. J., Berriman A. C., Butt R. D. et al* Influence of entrance channel properties on heavy-ion reaction dynamics // Eur. Phys. J. - 2002. - Vol. A13, No. 1 - 3. - P. 149 - 154.

24. *Hinde D. J., Dasgupta M.* Importance of entrance channel dynamics on heavy element formation // Nucl. Phys. A - 2004. - Vol. 734. - P. 148 - 155.
25. *Newton J. O., Butt R. D., Dasgupta M. et al.* Systematic failure of the Woods-Saxon nuclear potential to describe both fusion and elastic scattering: Possible need for a new dynamical approach to fusion // Phys. Rev. C - 2004. - Vol. 70. - P. 024605.
26. *Morton C. R., Berriman A. C., Dasgupta M. et al.* Coupled-channels analysis of the $^{16}O+^{208}Pb$ fusion barrier distribution // Phys. Rev. C - 1999. - Vol. 60. - P. 044608.
27. *Баррет Р., Джексон Д.* Размеры и структура ядер. - К.: Наук. думка, 1981. - 420 с.
28. *Siwek-Wilczynska K., Wilczynski J.* Empirical nucleus-nucleus potential deduced from fusion excitation functions // Phys. Rev. C - 2004. - Vol. 69. - P. 024611.
29. *Raman S., Nestor C. W., Tikkanen P.* Transition probability from the ground to the first-excited 2+ state of even-even nuclides // At. Data Nucl. Data Tabl. - 2001. - Vol. 78, No. 1. - P. 1 - 126.
30. *Kibedi T., Spear R. H.* Reduced electric-octupole transition probabilities, $B(E3;0_1^+\rightarrow 3_1^-)$ - an update // At. Data Nucl. Data Tabl. - 2002. - Vol. 80, No. 1. - P. 35 - 82.

ДИФФУЗНОСТЬ ЯДЕРНО-ЯДЕРНОГО ПОТЕНЦИАЛА И ДИФФУЗНОСТЬ РАСПРЕДЕЛЕНИЯ ПЛОТНОСТИ В ЯДРАХ

В. Ю. Денисов, В. А. Нестеров

Детально исследуются потенциалы взаимодействия ядер, вычисленные с помощью сил Скирма в рамках модифицированного приближения Томаса - Ферми и теории Хартри – Фока - БКШ. С ростом параметра диффузности распределения плотности у взаимодействующих ядер происходит уменьшение высоты барьера между ядрами, увеличение глубины ямы захвата и поперечного сечения слияния. Показано, что величина параметра диффузности ядерной части потенциала на больших расстояниях между ядрами, вычисленная с помощью сил Скирма, превышает величину параметра диффузности распределения плотности нуклонов во взаимодействующих ядрах приблизительно в 1,5 раза. Реалистические значения параметра диффузности ядерного взаимодействия между средними и тяжелыми ядрами лежат в интервале $a \approx 0{,}75$ - 0,90 фм.

ДИФУЗНІСТЬ ЯДЕРНО-ЯДЕРНОГО ПОТЕНЦІАЛУ ТА ДИФУЗНІСТЬ РОЗПОДІЛУ ГУСТИНИ В ЯДРАХ

В. Ю. Денисов, В. О. Нестеров

Детально досліджуються потенціали взаємодії ядер, обчислені за допомогою сил Скірма в рамках модифікованого наближення Томаса - Фермі і теорії Хартрі – Фока - БКШ. З ростом параметра дифузності розподілу густини взаємодіючих ядер має місце зменшення висоти бар’єра між ядрами, збільшення глибини ями захоплення та поперечного перерізу злиття. Показано, що величина параметра дифузності ядерної частини потенціалу на великих відстанях між ядрами, обчислена за допомогою сил Скірма, перевищую величину параметру дифузності розподілу густини нуклонів у ядрах, що взаємодіють, приблизно у 1,5 рази. Реалістичні значення параметра дифузності ядерної взаємодії між середніми та важкими ядрами лежать в інтервалі між $a \approx 0{,}75$ - 0,90 фм.