

RESONANT PRODUCTION OF ELECTRON-POSITRON PAIR BY A PHOTON WITH PHOTON EMISSION IN STRONG MAGNETIC FIELD

P. I. Fomin^{1,2}, R. I. Kholodov¹

¹ Institute of Applied Physics, National Academy of Sciences of Ukraine, Sumy, Ukraine

² Bogolyubov Institute of Theoretical Physics, National Academy of Sciences of Ukraine, Kyiv, Ukraine

The quantum-electrodynamical process of electron-positron pairproduction is considered at the collision of fast heavy nuclei in a region between them, where the magnetic fields of nuclei are added, and their Coulomb fields are mutually compensated. In that region the magnetic fields of nuclei can achieve a value 0^{12} Gs, in the case when the size of region has the order of magnitude of Compton wave-length of electron 10^{-11} cm. The process is accompanied by emitting of final photon. A kinematics and resonance conditions are found in approximation of the strong magnetic field and low-excited states of electron and positron (ultraquantum approximation). The resonance conditions are realized, if energy of the final photon is equal to the distance between Landau levels of an electron. Probability of the resonance electron-positron pairproduction is found in form of Breit-Wigner crossection.

1. Introduction

The quantum-electrodynamics QED processes in the presence of strong magnetic field close to the critical value of about 10^{13} Gs may accompany fast heavy nuclei collisions. The magnetic field is generated by two colliding nuclei, which play role of two current. The magnetic field produced by colliding nuclei in the region between them at the moment of the closest approach has order of magnitude about 10^{12} Gs in the case, when that region has the size of Compton wavelength of electron (Fig. 1). The electric fields of nuclei mutually compensate one another in that region.

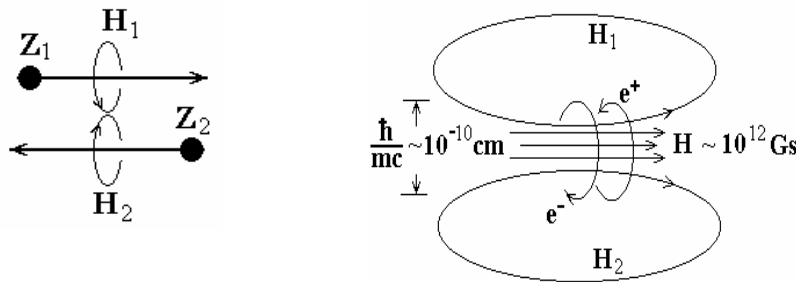


Fig. 1. Magnetic field of colliding nuclei and e^+e^- pair in that field.

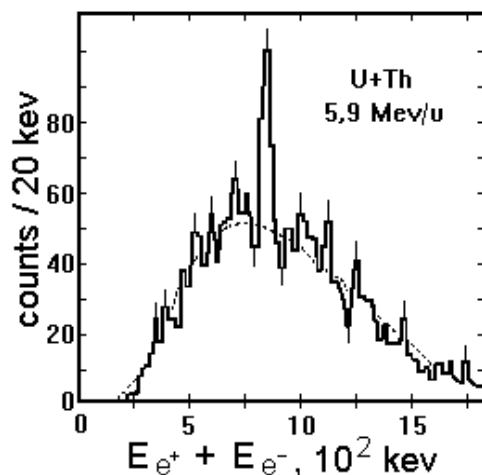


Fig. 2. Narrow peaks in heavy ions collision experiments at GSI, Darmstadt [1, 2] are the resonant pair production on the Landau levels:

$$E_n = E_{e^+} + E_{e^-} = 2\sqrt{m^2 + 2neH + \pi^2 / L_z^2}.$$

We consider, that the series of quasi-equidistant narrow peaks in the electron-positron distribution of total energy, observed more than ten years ago in heavy ions collision at GSI, Darmstadt [1, 2], is a result of movement of an electron-positron pair in such magnetic field in that region (Fig. 2). Narrow lines are the resonant pair production on the Landau levels.

The first theoretical works for study of the process of electron-positron pair photoproduction in magnetic field were performed in the middle of the last century yet [3]. This process in the field of more complicated configuration (magnetic field plus plane wave along field) is studied in paper [4]. Later two-photon electron-positron pairproduction in the magnetic field was studied [5]. However, it should be noted, that similar quantum-electrodynamics processes can be accompanied by emitting of additional photon. This work is devoted to the study of such a process. In this work we use the relativism system of units: $\hbar = 1$, $c = 1$.

2. Kinematics of the process

The examined process is described by the Feynman diagrams represented in Fig. 3, where wavy lines present photons, and the continuous lines correspond to electrons (positrons) with 4 momenta $\mathbf{p} = (\varepsilon, 0, 0, p)$, $\mathbf{p}^+ = (\varepsilon^+, 0, 0, p^+)$ respectively [6, 7]. The wave function of electron has the form [8, 9]:

$$\Psi^- = \frac{1}{\sqrt{S}} e^{-i(\varepsilon t - p_y \cdot y - p \cdot z)} \Psi^-(\zeta), \quad \zeta = \sqrt{hm}(x + p_y / hm^2), \quad h = H/H_0 = eH/m^2, \quad (1)$$

where $\Psi^-(\zeta)$ is bispinor, which is expressed through Hermits functions. For the electron and positron the following laws of dispersion are performed in the magnetic field:

$$\varepsilon = \sqrt{\tilde{m}^2 + p^2}, \quad \varepsilon^+ = \sqrt{m^{*2} + (p^+)^2}, \quad \tilde{m}^2 = m^2 + 2l^- hm^2, \quad m^{*2} = m^2 + 2l^+ hm^2, \quad (2)$$

where l^-, l^+ are Landau levels of electron and positron.

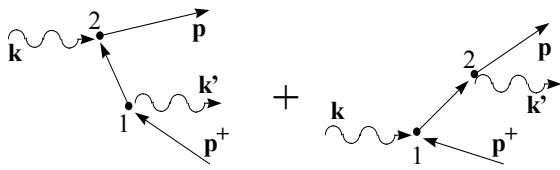


Fig. 3. Feynman diagrams for the process of electron-positron pair photoproduction with emitting of a photon.

The function (1) has the form of a flat wave in relation to three variables namely t, y, z , therefore amplitude of the process contains three Dirac delta functions, which correspond to the laws of conservation as follows:

$$\omega = \omega' + \varepsilon + \varepsilon^+, \quad k_z = k_z' + p + p^+, \quad k_y = k_y' + p_y + p_y^+. \quad (3)$$

The analysis of these expressions determines the kinematics of the process [10]. For this purpose it is convenient to define the function $f(p)$, which for the real processes applies in a zero

$$f(p) = \omega - \omega' - \sqrt{\tilde{m}^2 + p^2} - \sqrt{m^{*2} + (p + \omega' u - \omega \cdot v)^2}, \quad v = \cos \theta, \quad u = \cos \theta'. \quad (4)$$

Dependence of this function f on the longitudinal momentum of electron p is represented on Fig. 4.

As follows from a picture, the process of electron-positron pair production by a photon can take place only at energies of a photon higher than some threshold value. This value depends on frequency of a final photon and is defined by following expression: $\partial f / \partial p = 0, f = 0$.

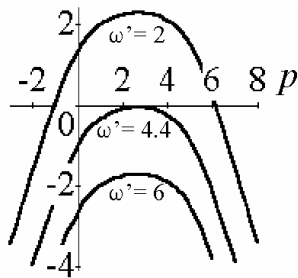


Fig. 4. Dependence $f(p)$ at the different values of frequency of a final photon. For $h = 0.3, l^- = 2, l^+ = 1, \omega = 10, v = 0.5, u = 0$.

The thresholds values of energies and momenta of particles are equal thus:

$$\varepsilon_m = \frac{\tilde{m}}{\sqrt{1 - \beta^2}} = \frac{\tilde{m}}{\tilde{m} + m^*} W, \quad p_m = \beta \cdot \varepsilon_m, \quad \varepsilon_m^+ = \frac{m^*}{\sqrt{1 - \beta^2}} = \frac{m^*}{\tilde{m} + m^*} W, \quad p_m^+ = \beta \cdot \varepsilon_m^+, \quad (5)$$

where $W \equiv \omega - \omega', \beta = \frac{k_z - k_z''}{\omega - \omega''}$.

The values of energies and momenta of an electron (positron) above threshold have the form

$$\varepsilon_{1,2} = \frac{a \pm b \cdot \beta}{2W(1-\beta^2)}, \quad p_{1,2} = \frac{a \cdot \beta \pm b}{2W(1-\beta^2)}, \quad \varepsilon_{1,2}^+ = \frac{a^+ \mp b \cdot \beta}{2W(1-\beta^2)}, \quad p_{1,2}^+ = \frac{a^+ \cdot \beta \mp b}{2W(1-\beta^2)}, \quad (6)$$

where $a = W^2(1-\beta^2) + \tilde{m}^2 - m^{*2}$, $a^+ = W^2(1-\beta^2) - \tilde{m}^2 + m^{*2}$, $b^2 = a^2 - 4\tilde{m}^2W^2(1-\beta^2)$.

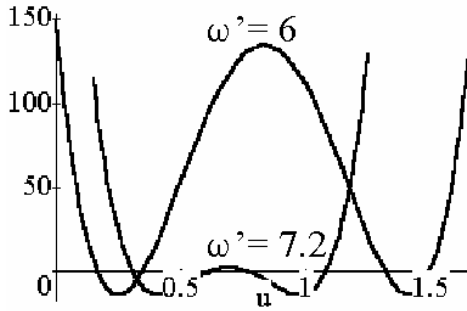


Fig. 5. Dependence $b(p)$ at the different values of frequency of a final photon. For $h = 0.3$, $l^- = 2$, $l^+ = 1$, $\omega = 10$, $v = 0.5$.

The final photon is radiated with frequency in interval $0 \leq \omega' \leq \omega - (\tilde{m} + m^*)$, ($\beta = 0$), besides the angle of radiation of a photon is limited by an interval which is defined by following condition $p_1 = p_2$ or $b(u) = 0$. The dependence of $b(u)$ is presented in Fig. 5. Crossing of lines with abscises axis in Fig. 5. corresponds to the limit values of cosines of angles, that have the form

$$u_{\min} = \frac{\omega v}{\omega'} - \frac{1}{\omega'} \sqrt{W^2 - (\tilde{m} + m^*)^2}, \quad u_{\max} = \frac{\omega v}{\omega'} + \frac{1}{\omega'} \sqrt{W^2 - (\tilde{m} + m^*)^2}. \quad (7)$$

In common case a final photon is radiated under an angle $u_{\min} \leq u \leq u_{\max}$, $e \cdot u_{\min} > -1$, $u_{\max} < 1$. In the examined process the impulse of the electron also depends on the direction of movement of the photon. Such dependence is represented in Fig.6.

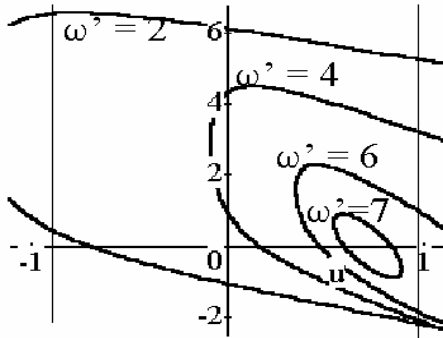


Fig. 6. Dependence of momentum of electron on the direction of movement of the photon at the different values of frequency of final photon. For $h = 0.3$, $l^- = 2$, $l^+ = 1$, $\omega = 10$, $v = 0.5$.

3. Probability of the process

We examine a case of strong field, when the individual energy levels are experimentally distinguishable. Thus the number of final states of electron is equal to one. This approximation is called ultra-quantum approximation. However, the magnetic field that is 10^{12} Gs in units critical field is a good small parameter of the problem. Due to the presence of the small parameter all calculations can be made analytically and it is possible to receive in Furry picture the simple expressions for probabilities of the process.

The amplitude of the process is defined by standard rules of Furry picture of QED according to Feynman diagrams Fig. 3.

$$S_{if} = -e^2 \int dx_1 d^3 r_1 \cdot dx_2 d^3 r_2 \times \bar{\Psi}^-(\mathbf{x}_1) \left[\hat{A}'(\mathbf{x}_1) \frac{1}{i} G_H(\mathbf{g}) \hat{A}(\mathbf{x}_2) + \hat{A}(\mathbf{x}_1) \frac{1}{i} G_H(\mathbf{f}) \hat{A}'(\mathbf{x}_2) \right] \Psi^+(\mathbf{x}_2).$$

This expression in ultraquantum approximation near the resonance condition can be transform to the following form

$$S_{if} = \frac{ie^2(2\pi)^4 \delta^3(k - k' - p^- - p^+)}{2VS\sqrt{m\omega'}} \sqrt{\hbar} e_z R \left(\frac{-e^{i\Phi_g} \Pi^{'+}}{\omega' - \omega_r + i\frac{\Gamma_g}{2}} + \frac{e^{i\Phi_f} \Pi^{'-}}{\omega' - \omega_r + i\frac{\Gamma_f}{2}} \right) \quad (8)$$

Here $\Pi^{\pm} = \cos\theta' \cos\alpha' \pm i \sin\alpha' e^{-ib'}$ is a value that characterizes a polarization of the final photon by using parameter a' , b' ; Φ is a phase of the amplitude; Γ_g , Γ_f are the width's of resonance's, witch are determined as the total probability of decay of intermediate state, namely the probability of magnetobremstrahlung.

The resonance conditions are realized, if energy of the final photon is equal to the distance between Landau levels of an electron $\omega' = \omega_r = (n - l^-) \hbar m$.

The square of modules of the amplitude (8) gives the probability

$$S_{if} = M \frac{1}{SV} \delta^3, \quad W_{if} = |M|^2 \frac{1}{S^2 V^2} \cdot \frac{\delta^3 TS}{(2\pi)^3}, \quad (9)$$

where δ^3 are three Dirac delta functions (3). The number of final states is

$$dN = \frac{S d^2 p \cdot S d^2 p^+ \cdot V d^3 k'}{(2\pi)^7}. \quad (10)$$

Product of the probability (9) and the number of final states (10) gives differential probability.

It should be noted that the physical values don't must have any nonphysical constants such as L_x , L_y , L_z . It has place for differential probability if it takes into account, that $L_x = 2|x_0| = 2|p_y| / \hbar m^2$. It is possible to present the differential probability of process in time unit through amplitude M in a next form [8, 9]

$$\frac{dW_{if}}{T} = \hbar m^2 |M|^2 \delta(\varepsilon) \frac{dp \cdot d^3 k'}{2(2\pi)^{10}}. \quad (11)$$

After integration over energy of particles the probability from amplitude of the first diagram Fig. 3 has the form of Breit - Wigner crosssection

$$\frac{dW}{d\omega' d\Omega'} = \frac{1}{(2\pi)^2} \cdot \frac{W_{pr.e^+} \cdot dW_{rad.e^-} / du}{((\omega' - \omega_r)^2 + n^2 \Gamma^2 / 4)}, \quad (12)$$

where $W_{pr.e^+}$ is a probability of pairproduction by the initial photon of final positron and intermediate electron; $dW_{rad.e^-} / du$ is a probability of the cyclotron radiation of the intermediate electron with transition into the final state. For the pairproduction into ground state by nonpolarized initial photon in magnetic field $h = 0.1$ or $H = 10^{12}$ Gs

$$W_{pr} = \alpha \hbar m^2 \exp(-2/h) / p \sim 10^{10} [1/c], \quad W_{rad} = n \alpha \hbar m^2 = n \Gamma \sim 10^{17} [1/c] \quad (13)$$

Then total probability of pairproduction with additional final photon is equal

$$W = \int_0^{\omega'_{\max}} d\omega' \int_{u_{\min}}^{u_{\max}} du \left(\frac{dW(p_1)}{d\omega' du} + \frac{dW(p_2)}{d\omega' du} \right) = W_{pr}. \quad (14)$$

This probability at resonance conditions is equal to the probability of photon pairproduction without emitting of a photon.

The authors are grateful to V. E. Storizhko for his constant help in holding this work and also to S. P. Roshchupkin for useful discussions.

The work was partly supported by the Grant for the young researchers of the National Academy of Sciences of Ukraine.

REFERENCES

1. *Koenig I., Berdermann E., Bosch F. et al.* Investigations of correlated e^+e^- emission in heavy-ion collisions near the Coulomb barrier // *Z. Phys. A.* - 1993. - Vol. 346. - P. 153 - 164.
2. *Bar R., Balanda A., Baumann J. et al.* Experiments on e^+e^- -line emission in HI collisions // *Nuclear Phys.* - 1995. - Vol. 583. - P. 237 - 145.
3. *Klepikov N.P.* Photons and electron-positron pairs emission in magnetic field // *Zh. Eksp. Teor. Fiz.* -1954. - Vol. 26. - P. 19 - 34 (in Russian).
4. *Oleynik V.P.* Production of electron-positron pair by a photon in field of electromagnetic wave and homogeneous magnetic field // *Zh. Eksp. Teor. Fiz.* - 1971. - Vol. 61. - P. 27 - 35 (in Russian).
5. *Kozlenkov A.A., Mitrofanov I.G.* Twophoton production of electron-positron pair in strong magnetic field // *Zh. Eksp. Teor. Fiz.* - 1986. - Vol. 91. - P. 1978 - 1989 (in Russian).
6. *Sokolov A.A., Ternov I.M.* Relativistic electron. - Moscow: "Nauka", 1974. - 392 p. (in Russian).
7. *Akhiezer A.I., Berestetskiy V.B.* Quantum Electrodynamics. - 4th edition. - Moscow: "Nauka", 1981; New York: "Wiley", 1965.
8. *Fomin P.I., Kholodov R.I.* Resonance Compton scattering in an external magnetic field // *JETP.* - 2001. - Vol. 90. - P. 281; *Zh. Eksp. Teor. Fiz.* - 2001. - Vol. 117. - P. 319 - 325 (in Russian).
9. *Fomin P.I., Kholodov R.I.* Resonance double magnetic bremsstrahlung in a strong magnetic field // *JETP.* - 2003. - Vol. 96. - P. 315; *Zh. Eksp. Teor. Fiz.* - 2003. - Vol. 123. - P. 356 - 361 (in Russian).
10. *Fomin P.I., Kholodov R.I.* Photoproduction of the electron-positron pair with photon emission kinematics in strong magnetic field // *Prob. Atom. Scien. Tech.* - 2005. - No. 6. - P. 43 - 45.

РЕЗОНАНСНОЕ РОЖДЕНИЕ ЭЛЕКТРОН-ПОЗИТРОННОЙ ПАРЫ С ИЗЛУЧЕНИЕМ ФОТОНА В СИЛЬНОМ МАГНИТНОМ ПОЛЕ

П. И. Фомин, Р. И. Холодов

Рассмотрен квантово-электродинамический процесс рождения электрон-позитронной пары при столкновении быстрых тяжелых ядер в области между ними, где магнитные поля ядер складываются, а их кулоновские поля взаимно компенсируются. В этой области магнитные поля ядер могут достигать величины 10^{12} Гс в случае, когда размер области имеет порядок величины комптоновской длины волны электрона 10^{-11} см. Процесс сопровождается испусканием конечного фотона. Найдены кинематика и резонансные условия в приближении сильного магнитного поля и слабозбужденных состояний электрона и позитрона (ультраквантовое приближение). Резонансные условия реализуются, если энергия конечного фотона равняется расстоянию между уровнями Ландау электрона. Получена вероятность резонансного рождения e^+e^- -пары в виде сечения Брейта - Вигнера.

РЕЗОНАНСНЕ НАРОДЖЕННЯ ЕЛЕКТРОН-ПОЗИТРОННОЇ ПАРИ З ВИПРОМІНЮВАННЯМ ФОТОНА В СИЛЬНОМУ МАГНІТНОМУ ПОЛІ

П. І. Фомін, Р. І. Холодов

Розглянуто квантово-електродинамічний процес народження електрон-позитронної пари при зіткненні швидких важких ядер в області між ними, де магнітні поля ядер додаються, а їх кулонівські поля взаємно компенсуються. У цій області магнітні поля ядер можуть досягати величини 10^{12} Гс у випадку, якщо розмір області має порядок величини комптонівської довжини хвилі електрона 10^{-11} см. Процес супроводжується випромінюванням кінцевого фотона. Знайдено кінематику й резонансні умови в наближенні сильного магнітного поля та слабо збуджених станів електрона й позитрона (ультраквантове наближення). Резонансні умови реалізуються, якщо енергія кінцевого фотона дорівнює відстані між рівнями Ландау електрона. Здобуто вираз для імовірності резонансного народження e^+e^- -пари у вигляді перерізу Брейта - Вігнера.