

ABOUT ALTERNATIVE APPROACH TO ANALYTIC REPRESENTATION OF THE COULOMB BREAKUP AMPLITUDE

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The renewed method of accounting of the contributions to the breakup reaction amplitude due to Coulomb interaction in the external spatial region, that takes into account the corrections of the first and the second orders due to the finiteness of radius of interaction of two clusters which form a decaying projectile particle, is proposed. The application of this method as a component part of the coupled channel method with discretization of continuum would match reduce the required model space as well as the number of coupled equations to be solved. It is shown that an approach not using the method of infinitesimal linear-fractional transformations for algebraic presentation of the main part of amplitude of Coulomb desintegration as a re-sult gives more cumbersome expressions, than with the use of this method. Thus, as a result of double emp-loying of indicated transformations to the integral for the Coulomb desintegration amplitude which is defined in zero-radius approximation, algebraic expression of the simplest form is derived.

1. Introduction

In our recent work [1] a new method of calculation of the amplitude of the direct Coulomb breakup in two charged particles was proposed. This method uses the zero range approximation for the interaction of clusters that form a projectile particle and takes into account the corrections of the first and the second orders to that approximation. We intend to incorporate it in the method of con-tinuum discretized coupled channels (CDCC) for simplifying the calculus.

The method of CDCC has been successful in describing nuclear reactions including weakly bound projectiles [2 - 9, 14]. CDCC has been attracting much attention since the advent of experiments with radioactive beams, because projectile breakup processes are essential to many of such reactions. CDCC plays an important role in the spectroscopic studies of radioactive nuclei through the nuclear reactions involving such nuclei.

It has been shown that CDCC is the first-order approximation to the distorted Faddeev equations, and corrections to the converged CDCC solution are negligible within the region of space in which the reaction takes place [10]. In CDCC the continuum of momentum k states of relative motion of the projectile desintegration fragments is exposed to discretization on finite interval $[0, k_{max}]$ for every relative angular momentum l , when $l \in [0, l_{max}]$, at interval $[0 \leq r \leq r_{max}]$ of relative radial variable. Then the system of integro-differetial coupled equations for the waves functions $\Psi_{iLL}(K_i, R)$, which describe the scattering of the center of masses of desintegrating projectile with a target nucleus is to be solved. As for the discretization methods, three ones have been proposed so far: the average (Av) [2 - 4, 11], the midpoint (Mid) [4, 12], and the pseudo-state (PS) [2, 13] methods.

From a practical point of view, however, CDCC calculation including long-ranged Coulomb coupling-potentials requires extremely large model space. For example, to determine $S_{17}(0)$ -factor from an analysis of $^{208}\text{Pb}(^8\text{B}, p + ^7\text{Be})^{208}\text{Pb}$ at 52 MeV/nucleon [14] it has required the resulting number of scattering-channels of 138 and the maximum values of r , R , and L has been, respectively, 200 fm, 1000 fm, and 12000.

More over, typically the number of partial waves amounts to 15000 for the MSU data [15]. Although interpolation technique for angular momentum reduces the number of CC equations to be solved in terms of J , those with huge angular momenta are rather unstable and careful treatment is necessary. In this sense, it seemed almost impossible to apply CDCC to the GSI data at the energy 250 MeV/nucleon [16], where J_{max} is expected to exceed 100000.

In order to treat both Coulomb and nuclear breakup processes at intermediate energies with high accuracy and computational speed, a new method was proposed in Ref. [17], namely, a hybrid calculation with the three-body CDCC method and the eikonal-CDCC (E-CDCC) method. E-CDCC describes the center-of-mass motion of the projectile relative to the target by straight-line approxi-mation (or by using Coulomb wave functions instead of plane waves) and treats the excitation of the projectile explicitly by CDCC with the momentum-bin method or the PS method. E-CDCC drastically reduces computation time and eliminates many problems concerned with huge angular mo-mentum in solving coupled-channel equations.

In our proposing variant of the method CDCC the model space on the variable R and on the angular momentum L is limited to the largest radius $R_{\max} \gg \max(r_{\max}, R_{\text{nucl}})$ and to the value of $K_{\max} R_{\max}$ for the scattering of the chosen loosely bound projectile (or ejectile in two step reactions). Here R_{nucl} is the largest radius of the nuclear interaction, K_{\max} corresponds to the scattering energy in the center of masses. The model space on the relative variable of r , on the relative momentum of k and on the relative angular momentum of ℓ of particles from the breakup assumes to be of such, what is required for discretizing in the internal region $[0, r_{\max}]$ of enough number of desintegration channels.

In an internal region the partial breakup amplitudes are calculated with the aid of one of three aforementioned methods of the continuum discretization and of solving of the coupled discrete channels equations. In an external region the DWBA-approaching is used to determine contributions of the purely Coulomb breakup by the method of Ref. [1], contributions of six types of amplitudes taking into account nuclear-Coulomb interference, and of yet one amplitude of nuclear-nuclear interference in the region of Coulomb interaction. As a formula for the main part of the Coulomb breakup amplitude in [1] and for corrections of the 1-st order to such amplitude embrace an integration over hole internal region of R , then expansions of these amplitudes over composite spherical basis must be subtracted, after discretizing on the same basis, from the amplitude of breakup, found by means of the CDCC method. The corrections of the second and higher orders to the Coulomb breakup amplitude it is possible to calculate by the straight integration in suitable curvilinear space coordinates. Certainly, one could somewhat amount R_{\max} and L_{\max} for decreasing contributions from an external region and for improving an exactness of calculation of aforementioned corrections. In the next sections starting from the DWBA expression for purely Coulomb breakup amplitude we shall find the analytical approximation for that amplitude and shall investigate its main properties.

2. Approximations for purely Coulomb breakup amplitude

We assume that a projectile nucleus is clustered and is in the ground state with a zero orbital momentum of the clusters relative motion. We assume also, that its wave function and wave functions, which describe the motion of each cluster relative to the projectile center of masses by a form are close to the own functions of the Shrödinger equation with oscilaton potentials frequencies of which are identical. Then in the cluster expansion of the projectile own function it is enough to take into account only the first member. In the distorted-wave approximation we also neglect here the polarizing potential addition $[V_b(r_b) - V_b(R_b/\delta)]$ to the interaction in the final state in the expression for the internal matrix element. Then the transition matrix for the reaction of elastic breakup $A(c, ba)A$ is represented in the following form:

$$T_{fi} = N_{c \rightarrow ba} \lim_{\omega \rightarrow 0} \left(\iint e^{-\omega R} \Psi^{(-)}(\vec{k}_a, \vec{r}_a)^* \Psi^{(-)}(\vec{q}_b, \vec{R}_b)^* \times \right. \\ \left. \times \langle \varphi_b(\xi_1) \varphi_a(\xi_2) | V_{ba}(\vec{r}, \xi_1, \xi_2) | \phi_c(\vec{r}, \xi_1, \xi_2) \rangle \Psi^{(+)}(\vec{q}_c, \vec{R}) d^3 r d^3 R \right) \quad (1)$$

In the integrand of an expression (1) functions $\varphi_b(\xi_1)$ and $\varphi_a(\xi_2)$ are antisymmetric own wave functions of the ground states of ejected nuclei b and a , $\phi_c(\vec{r}, \xi_1, \xi_2)$ is an antisymmetric own wave function of a projectile c , $V_{ba}(\vec{r}, \xi_1, \xi_2)$ is a total interaction of particles a and b in the final state of the reaction; it is assumed to be equal to an interaction between corresponding clusters bounded in a projectile c ; $V_b(r_b)$ is a total interaction of a particle b with a target nucleus t . The coefficient $N_{c \rightarrow ba}$ is the decaying cluster amplitude. Upon integrating in the expression (1) at first the folding procedure with respect to internal coordinates ξ_1 and ξ_2 of the ejected particles has to carry out.

Here for the input channel the relative Jakobi coordinates are chosen to be $\{\vec{r}, \vec{R}\}$, where $\vec{r} = \vec{r}_b - \vec{r}_a$, $\vec{R} = -\vec{R}_t$, and for the output channel the systems of pairs $\{\vec{r}_a, \vec{R}_b\}$ and $\{\vec{r}_b, \vec{R}_a\}$ will be used. A vector \vec{r}_a is directed from the center of masses (c.m.) of a target t to c. m. of a particle a , and a vector \vec{R}_b is directed from c. m. of a particle b to c. m. of the subsystem $(a+t)$. In the similar manner a vector \vec{R}_t is directed from a target nucleus t to c. m. of the subsystem $(a+b)$.

The dimension is chosen such that $\hbar = c = 1$.

If one takes into account solely the Coulomb breakup and accepts that all particles involving in the reaction have electric charges, all the functions Ψ in the expression (1) come to the functions describing the motion of each of particles a, b, c in the Coulomb field of a target nucleus t . For example, the wave function of a projectile c can be expressed [18] in terms of the degenerate Kummer hypergeometric [19, 20] function

$$\Psi^{(+)}(\vec{q}_c, \vec{R}) = A(\eta_c) e^{i(\vec{q}_c \cdot \vec{R})} \Phi(-i\eta_c; 1; iz_c(\vec{R})). \quad (2)$$

Here the following notations are introduced:

$$A(\eta_c) = e^{-\pi\eta_c/2} \Gamma(1+i\eta_c), \quad z_c(\vec{R}) = q_c R - (\vec{q}_c \cdot \vec{R}), \quad \eta_c = \frac{M_c M_t e^2 (Z_a + Z_b) Z_t}{(M_c + M_t) q_c}. \quad (3)$$

The wave functions of the output channel are expressed [18] through respective wave functions with an asymptotic in a form of a sum of the incident plain wave, distorted by the Coulomb interaction, and of the incoming waves. These relations can be written for complex conjugated functions in the following form:

$$\Psi^{(-)}(\vec{k}_a, \vec{r}_a)^* = \Psi^{(+)}(-\vec{k}_a, \vec{r}_a), \quad \Psi^{(-)}(\vec{q}_b, \vec{R}_b)^* = \Psi^{(+)}(-\vec{q}_b, \vec{R}_b). \quad (4)$$

The Sommerfeld parameters in these functions are determined as follows:

$$\eta_a = \frac{M_a M_t e^2 Z_a Z_t}{(M_a + M_t) k_a}, \quad \eta_b = \frac{M_b M_t e^2 Z_b Z_t}{(M_a + M_b + M_t) q_b}. \quad (5)$$

Further, like a manner of Ref. [1] we substitute a function (2) and functions (4), expressed in the same form, in the integrand in the right-hand side of (1). As we do so we use the following relations between the Jacobi coordinates in the input and breakup channels

$$\vec{r}_a = \vec{R} - \gamma \vec{r}, \quad \vec{R}_b = \delta \vec{R}_x, \quad \vec{R}_x = \vec{R} + \xi \vec{r}, \quad (6)$$

were

$$\delta = \frac{M_t}{M_a + M_t}, \quad \gamma = \frac{M_b}{M_a + M_b}, \quad \xi = \frac{M_a (M_a + M_b + M_t)}{M_t (M_a + M_b)}. \quad (7)$$

In the result of changing of the spatial coordinate in hypergeometric part of the wave function of the particle b this part will depend on the following parameters:

$$\vec{p}_b = \delta \vec{q}_b, \quad \vec{z}_x(\vec{R}_x) = p_b R_x + (\vec{p}_b \cdot \vec{R}_x), \quad \eta_b = \frac{M_b M_t^2 e^2 Z_b Z_t}{(M_a + M_t)(M_a + M_b + M_t) p_b}. \quad (8)$$

Moreover, we introduce the following denotations for the particle momenta relative to the center of masses of the three body system ($a + b + t$), as well as a denotation for a product of three Sommerfeld parameters

$$\vec{q}_f = \vec{q}_a + \vec{q}_b, \quad \vec{q} = \vec{q}_c - \vec{q}_f, \quad \vec{k} = \frac{M_a \vec{q}_b - M_b \vec{q}_a}{M_a + M_b}, \quad A(\eta_a, \eta_b, \eta_c) = A(\eta_a) A(\eta_b) A(\eta_c). \quad (9)$$

To transform a product of the plane waves in the integrand in the expression (1) we use the following relations, which connect with each other sums of scalar products of particle momenta and their conjugate coordinates in different Jacobi coordinate systems,

$$(\vec{k}_a \vec{r}_a) + (\vec{q}_b \vec{R}_b) = (\vec{k}_b \vec{r}_b) + (\vec{q}_a \vec{R}_a) = (\vec{k} \vec{r}) + (\vec{q}_f \vec{R}). \quad (10)$$

In the result we carry out the following representation for the transition matrix

$$T_{f_1(\omega)} = A(\eta_a, \eta_b, \eta_c) N_{c \rightarrow ba} \lim_{\omega \rightarrow 0} \left[\int e^{-\omega R + i(\vec{q} \cdot \vec{R})} \Phi(-i\eta_c; 1; iz_c(\vec{R})) \times \right. \\ \left. \times \int e^{-i(\vec{k} \cdot \vec{r})} \langle \varphi_b \varphi_a | V_{ba}(r) | \phi_c(r) \rangle \Phi(-i\eta_a; 1; i\tilde{z}_a(\vec{r}_a)) \Phi(-i\eta_b; 1; i\tilde{z}_x(\vec{R}_x)) d^3 r d^3 R \right]. \quad (11)$$

Now, making use well known exponential operators in the internal integral, we expand both hypergeometrical functions, which depend on coordinates \vec{r}_a and \vec{R}_x , with respect to powers of their deflections from a vector \vec{R} . Further, as differential operators in the exponents operate on different variables and so they commute, we combine a product of exponential operators into one with the following total exponent:

$$-(\vec{r} \cdot \vec{p}) = -\vec{r}(\gamma \nabla_a - \xi \nabla_x). \quad (12)$$

As a result we derive the following integral form for a total differential operator:

$$f_T(\vec{k} - i\vec{p}) = \int e^{-i(\vec{k} \cdot \vec{r})} \langle \varphi_b \varphi_a | V_{ba}(r) | \phi_c(r) \rangle e^{-i(\vec{r} \cdot \vec{p})} d^3 r. \quad (13)$$

Operators of gradients ∇_a and ∇_x act on arguments \vec{r}_a and \vec{R}_x of hypergeometric functions, respectively, and after that these arguments are put to be equal to the vector \vec{R} . Expanding now the second exponential function in powers of its exponent to the second order inclusive and integrating term by term a sum of integrals, we obtain an approximate representation of the operator (13). This representation takes into account up to the second order the corrections due to a finite interaction radius between the clusters. We mark that Kummer hypergeometric functions do not depend on the momentum \vec{k} of relative motion of the clusters. Therefore we can write the result as follows:

$$f_T(\vec{k} - i\vec{p}) = f_0(k) - i(\nabla_k f_0(k) \cdot \vec{p}) - \frac{1}{2}(\Delta_k f_0(k)) p^2 - \frac{1}{2}(\vec{p} \cdot \nabla_k)^2 f_0(k). \quad (14)$$

We shall mark that if corrections of the first and the second orders (from an external region) are taken into account, it is necessary to take into account also contributions from the Coulomb part of polarizing potential $[V_b(r_b) - V_b(R_b/\delta)]$. As a result we will obtain the polarizing addition to breakup amplitude in a form of the integral over an external region, involving the result of action on the wave functions of ejecting particles with the following differential operator:

$$f_{pol}(\vec{k}, \vec{p}) = \int e^{-i(\vec{k} \cdot \vec{r})} \langle \varphi_b \varphi_a | V_{pol}(\vec{r}, \vec{R}) | \phi_c(r) \rangle e^{-i(\vec{r} \cdot \vec{p})} d^3 r.$$

$$V_{pol}(\vec{r}, \vec{R}) = e^2 Z_b Z_t \left[\frac{1}{|\vec{R} + (1-\gamma)\vec{r}|} - \frac{1}{|\vec{R} + \xi\vec{r}|} \right].$$

This contribution is especially important if a target nucleus has not very much large atomic weight.

A function $f_0(k)$ is expressed through an integral of the product of the spherical Bessel function and the decay formfactor:

$$f_0(k) = 4\pi \int_0^\infty r^2 j_0(kr) \langle \varphi_b \varphi_a | V_{ba}(r) | \phi_c(r) \rangle dr. \quad (15)$$

Using explicit expressions for arguments $\tilde{z}_a(\vec{r}_a)$ and $\tilde{z}_x(\vec{R}_x)$, in the expression for the result of action of operator \vec{p} on a product of hypergeometrical functions we replace gradients with respect to coordinates \vec{r}_a and \vec{R}_x by operators of gradients with respect to momenta \vec{k}_a and \vec{R}_x , respectively. Then, taking into consideration the first two addends in (14) and using formulas (16), (17) from Ref. [1], the transition matrix of the Coulomb breakup, which takes into account the first-order corrections due to a finite interaction radius, can be written as follows:

$$T_{fi(\omega)} = A(\eta_a, \eta_b, \eta_c) N_{c \rightarrow ba} \left[\lim_{\omega \rightarrow 0} (-f_0(k)) \frac{\partial}{\partial \omega} - i(\nabla_k f_0(k)) \cdot [\gamma k_a (\vec{k} \cdot \nabla_{k_a}) - \xi p_b (\vec{k} \cdot \nabla_{p_b})] \right] X_{0,\omega}, \quad (16)$$

$$X_{0,\omega} = \int \frac{1}{R} e^{-\omega R + i(\vec{q} \cdot \vec{R})} \Phi(-i\eta_a; 1; i\tilde{z}_a(\vec{R})) \Phi(-i\eta_b; 1; i\tilde{z}_b(\vec{R})) \Phi(-i\eta_c; 1; i\tilde{z}_c(\vec{R})) d^3 R. \quad (17)$$

3. Alternative representation of the overlap integral

For integrating over a space of coordinate \vec{R} we use instead of each degenerate hypergeo-metric function in the integrand of (17) its representation in a form of a contour integral in the plane of complex variable

$$\Phi(-i\eta; 1; iz) = -\frac{1}{2\pi i} \int_1^{(0+)} e^{jzt} (-t)^{-1-i\eta} (1-t)^{i\eta} dt, \quad (18)$$

under the condition $Re(1+i\eta) > 0$. This relation directly ensues from the formula (6.11 (2)) of a book [20]. All powers interpret in the principal value sense, i.e. with the smallest absolute value of the argument of a base of a power function¹. The integration contour starts at the point $t=1$ and encircles the point $t=0$ in the positive direction.

Now we substitute three integral representations of the form (18) with proper indexes instead of the Kummer functions in the integrand of the expression (17), and then we change the integration orders (the four-fold integral converges because $\omega > 0$). After some simplifications we obtain as a result the overlap integral $X_{0,\omega}$ of such a form:

$$X_{0,\omega} = -\frac{i}{(2\pi)^3} \int_1^{(0+)} (-s)^{-1-i\eta_b} (1-s)^{i\eta_b} \int_1^{(0+)} (-t)^{-1-i\eta_a} (1-t)^{i\eta_a} \times \\ \times \int_1^{(0+)} (-\tau)^{-1-i\eta_c} (1-\tau)^{i\eta_c} \int \frac{1}{R} e^{-\omega R + i(\vec{K} \cdot \vec{R})} d^3 R d\tau dt ds. \quad (19)$$

The following denotations for linear combinations of wave numbers and wave vectors are here introduced

$$p = \omega - i(tk_a + sp_b + \tau q_c), \quad \vec{K} = \vec{q} + t\vec{k}_a + s\vec{p}_b - \tau\vec{q}_c. \quad (20)$$

After integrating of internal integral over volume in right-hand side of (19) we find, taking into account the relations (20), that the overlap integral (19) can be expressed in the following form:

$$X_{0,\omega} = -\frac{i}{2\pi^2} \int_1^{(0+)} (-s)^{-1-i\eta_b} (1-s)^{i\eta_b} \int_1^{(0+)} (-t)^{-1-i\eta_a} (1-t)^{i\eta_a} \times \\ \times \int_1^{(0+)} (-\tau)^{-1-i\eta_c} (1-\tau)^{i\eta_c} [a - cs - bt - d\tau + fst + hst + g\tau]^{-1} d\tau dt ds. \quad (21)$$

The values of the new parameters are the following:

$$a = q^2 + \omega^2, \quad b = -2(\vec{q} \cdot \vec{k}_a) + 2i\omega k_a, \quad c = -2(\vec{q} \cdot p_b) + 2i\omega p_b, \quad d = 2(\vec{q} \cdot \vec{q}_c) + 2i\omega q_c, \\ f = 2(\vec{k}_a \cdot \vec{p}_b) - 2k_a p_b, \quad g = -2(\vec{k}_a \cdot \vec{q}_c) - 2k_a q_c, \quad h = -2(\vec{p}_b \cdot \vec{q}_c) - 2p_b q_c. \quad (22)$$

¹ In Ref. [1] the words “of the argument” are missed mistakenly after “absolute value”.

We now perform in the three-fold integral (21) three linear-fractional transformations of the following forms:

$$\tau = \frac{\rho}{\rho\alpha - \alpha + 1}, \quad t = \frac{u}{u\beta - \beta + 1}, \quad s = \frac{v}{v\gamma - \gamma + 1}. \quad (23)$$

Similar replacements of integration variables were executed in Ref. [1], and the details of such transformations are expounded in it. Therefore here we bring at once a result over

$$\begin{aligned} X_{0,\omega} = & -\frac{i}{2\pi^2 a} (1-\alpha)^{i\eta_c} (1-\beta)^{i\eta_a} (1-\gamma)^{i\eta_b} \int_1^{(0+)} (-v)^{-1-i\eta_b} (1-v)^{i\eta_b} \int_1^{(0+)} (-u)^{-1-i\eta_a} (1-u)^{i\eta_a} \times \\ & \times \int_1^{(0+)} (-\rho)^{-1-i\eta_c} (1-\rho)^{i\eta_c} \left(1 - \frac{v(a\gamma-c)}{a(\gamma-1)} - \frac{u(a\beta-b)}{a(\beta-1)} - \frac{\rho(a\alpha-d)}{a(\alpha-1)} + \right. \\ & + \frac{uv(f-c\beta-b\gamma+a\beta\gamma)}{a(\beta-1)(\gamma-1)} + \frac{u\rho(g-b\alpha-d\beta+a\alpha\beta)}{a(\alpha-1)(\beta-1)} + \frac{v\rho(h-c\alpha-d\gamma+a\alpha\gamma)}{a(\alpha-1)(\gamma-1)} - \\ & \left. - \frac{uv\rho(f\alpha+h\beta+g\gamma-c\beta\alpha-b\gamma\alpha-d\beta\gamma+a\beta\gamma\alpha)}{a(\alpha-1)(\beta-1)(\gamma-1)} \right)^{-1} d\rho du dv. \end{aligned} \quad (24)$$

Further we choose free parameters α, β, γ such that corresponding coefficients of products $uv, u\rho, v\rho$ to be equal to zero. Then these parameters are determined as solutions of the following algebraic equations system:

$$\{f - c\beta - b\gamma + a\beta\gamma = 0, \quad g - b\alpha - d\beta + a\alpha\beta = 0, \quad h - c\alpha - d\gamma + a\alpha\gamma = 0\}. \quad (25)$$

Two sets of solutions look like:

$$\begin{aligned} \alpha \rightarrow & \frac{1}{a} \left(d \pm \sqrt{\frac{(bd-ag)(cd-ah)}{bc-af}} \right), \quad \beta \rightarrow \frac{1}{a} \left(b \pm \sqrt{\frac{(bc-af)(bd-ag)}{cd-ah}} \right), \\ \gamma \rightarrow & \frac{1}{a} \left(c \pm \sqrt{\frac{(cd-ah)(bc-af)}{bd-ag}} \right). \end{aligned} \quad (26)$$

For coefficients of remaining addends in the denominator of the integrand in (24) we shall use new denotations:

$$b_1 = \frac{(a\beta-b)}{(a\beta-a)}, \quad c_1 = \frac{(a\gamma-c)}{(a\gamma-a)}, \quad d_1 = \frac{(a\alpha-d)}{(a\alpha-a)}, \quad (27)$$

$$r_1 = \frac{f\alpha + h\beta + g\gamma - c\beta\alpha - b\gamma\alpha - d\beta\gamma + a\beta\gamma\alpha}{a(1-\alpha)(1-\beta)(1-\gamma)}.$$

As a result of the transformation of the integrand in the internal integral (with respect to ρ) in the right-hand side of (24), we find that this integral can be taken using a residue method. Assuming that a pole $(1-b_1u - c_1v)/(d_1 - r_1uv)$ is found to be out of integration contour, we find

$$-\int_1^{(0+)} \frac{(-\rho)^{-1-i\eta_c} (1-\rho)^{i\eta_c} \left(\rho - \frac{1-ub_1-vc_1}{d_1-r_1uv} \right)^{-1}}{(d_1-r_1uv)} d\rho = -2i\pi \frac{(1-d_1-ub_1-vc_1+uvr_1)^{i\eta_c}}{(1-ub_1-vc_1)^{1+i\eta_c}}. \quad (28)$$

Taking into account the equalities (25), (26), (27) in the integrand of (24), in the result of the substitution of (28) in (24) we obtain

$$X_{0,\omega} = -\frac{1}{\pi a} (1-\alpha)^{i\eta_c} (1-\beta)^{i\eta_a} (1-\gamma)^{i\eta_b} \int_1^{(0+)} (-v)^{-1-i\eta_b} (1-v)^{i\eta_b} \times \\ \times \int_1^{(0+)} (-u)^{-1-i\eta_a} (1-u)^{i\eta_a} (1-ub_1-vc_1)^{-1-i\eta_c} (1-d_1-ub_1-vc_1+uvr_1)^{i\eta_c} dv du. \quad (29)$$

Assuming that the following relations take place:

$$|b_1| < 1 \quad \text{and} \quad |b_1/(1-c_1)| < 1, \quad (30)$$

we carry out such an expansion in a Taylor series:

$$(1-vc_1-ub_1)^{-1-i\eta_c} = \sum_{n=0}^{\infty} \frac{(1+i\eta_c)_n}{n!} (-b_1)^n (-u)^n (1-vc_1)^{-1-n-i\eta_c}. \quad (31)$$

Substituting (31) in (29), we find

$$X_{0,\omega} = -\frac{1}{\pi a} (1-\alpha)^{i\eta_c} (1-\beta)^{i\eta_a} (1-\gamma)^{i\eta_b} \times \\ \times \sum_{n=0}^{\infty} \frac{(1+i\eta_c)_n}{n!} (-b_1)^n \int_1^{(0+)} \left[(-v)^{-1-i\eta_b} (1-v)^{i\eta_b} (1-vc_1)^{-n-1-i\eta_c} \times \right. \\ \left. \times \int_1^{(0+)} (-u)^{n-1-i\eta_a} (1-u)^{i\eta_a} (1-d_1-vc_1-ub_1+uvr_1)^{i\eta_c} du \right] dv. \quad (32)$$

To expand the third power function in the internal integral (with respect to variable u) we use the method of infinitesimal linear-fractional transformation. At first we perform in (32) the following linear-fractional transformations:

$$v = \frac{s}{1-(1-s)\delta}, \quad u = \frac{t}{1-(1-t)\lambda}. \quad (33)$$

As a result we obtain such a form of the representation of the overlap integral:

$$X_{0,\omega} = -\frac{1}{\pi a} ((1-\alpha)(1-d_1))^{i\eta_c} (1-\beta)^{i\eta_a} ((1-\gamma)(1-\delta))^{i\eta_b} \times \\ \times \sum_{n=0}^{\infty} \left[\frac{(1+i\eta_c)_n}{n!} (-b_1)^n (1-\lambda)^{-n+i\eta_a} \int_1^{(0+)} (-s)^{-1-i\eta_b} (1-s)^{i\eta_b} \left(1 - \frac{s\delta}{\delta-1} \right)^n \times \right. \\ \times \left(1 - \frac{s(\delta-c_1)}{\delta-1} \right)^{-n-1-i\eta_c} \int_1^{(0+)} (-t)^{n-1-i\eta_a} (1-t)^{i\eta_a} \left(1 - \frac{t\lambda}{\lambda-1} \right)^{-n-1-i\eta_c} \times \\ \left. \times \left(1 - \frac{s(c_1-\delta(1-d_1))}{(1-\delta)(1-d_1)} - \frac{t(b_1-\lambda(1-d_1))}{(1-\lambda)(1-d_1)} + \frac{st(-\delta b_1-\lambda c_1+\delta\lambda(1-d_1+r_1))}{(1-\delta)(1-\lambda)(1-d_1)} \right)^{i\eta_c} dt ds \right]. \quad (34)$$

We next choose the values of parameters λ and δ such that in the last power function in the integrand in (34) a coefficient of st is equal to zero, and a coefficient of s tends to zero in the limit:

$$\frac{c_1}{1-d_1} - \delta = \varepsilon \rightarrow 0.$$

Then dependences δ and λ on a infinitesimal parameter ε looks like:

$$\delta \rightarrow \frac{c_1}{1-d_1} - \varepsilon, \quad \lambda \rightarrow -\frac{b_1 c_1 + (d_1 - 1)r_1}{\varepsilon (d_1 - 1)^2} + \frac{b_1}{1-d_1}. \quad (35)$$

We denote a coefficient of the variable t in the same power function as f_1 . Then in the linear approximation on ε we obtain

$$f_1 = \frac{b_1 - \lambda(1-d_1)}{(1-\lambda)(1-d_1)} \rightarrow 1 + \varepsilon \frac{(1-d_1)(b_1 + d_1 - 1)}{b_1 c_1 - (1-d_1)r_1}. \quad (36)$$

Now at $t \neq 1$ the examined power function can be expanded in powers of a small parameter $s\varepsilon/((1-\delta)(1-f_1t))$. The result can be written as follows:

$$\left(1 - tf_1 - \frac{s\varepsilon}{1-\delta}\right)^{i\eta_c} = \sum_{m=0}^{\infty} \frac{(-1)^m (-i\eta_c)_m}{m!} (1-\delta)^{-m} \varepsilon^m (-s)^m (1-tf_1)^{-m+i\eta_c}. \quad (37)$$

Performing the substitution of the expansion (37) in the integral (34) and changing the orders of summation and integration, we integrate the series generated by the expansion (37) termwise. Then the expression (34) transforms to a double-fold series, the terms of which include the product of two contour integrals, either of the latter is integral representation (46) from Ref. [1] of the Appell function of two variables [20]. The integral representation (34) then becomes

$$\begin{aligned} X_{0,\omega} &= \frac{4\pi}{a} ((1-\alpha)(1-d_1))^{i\eta_c} (1-\beta)^{i\eta_a} ((1-\gamma)(1-\delta))^{i\eta_b} \times \\ &\times \lim_{\varepsilon \rightarrow 0} \sum_{n=0}^{\infty} \left(\frac{(1+i\eta_c)_n (-i\eta_a)_n}{n!n!} b_1^n (1-\lambda)^{-n+i\eta_a} \sum_{m=0}^{\infty} \left(\frac{(-i\eta_c)_m (-i\eta_b)_m}{m!m!} \times \right. \right. \\ &\times (1-\delta)^{-m} \varepsilon^m F_1 \left(m - i\eta_b; -n, n+1+i\eta_c; m+1; \frac{\delta}{\delta-1}, \frac{\delta-c_1}{\delta-1} \right) \times \\ &\left. \left. \times F_1 \left(n - i\eta_a; m - i\eta_c, n+1+i\eta_c; n+1; f_1, \frac{\lambda}{\lambda-1} \right) \right) \right). \end{aligned} \quad (38)$$

After substituting of the relations (35) and (36) in (38) and taking the limits at $\varepsilon \rightarrow 0$, we find, that, in force $f_1 \rightarrow 1$, only the second Appell function, the first argument of which becomes equal to 1, transforms to a Gauss hypergeometric function of ${}_2F_1$. Although the first Appell function, depending on a finite at $\varepsilon \rightarrow 0$ value of a parameter δ , does not transform to a Gauss function, it can be represented as a binomial sum with the growing upper limit n and with elements including Gauss functions. Thus, in place of two-fold series, as in Ref. [1], in this approach as a result we obtain the following three-fold series:

$$\begin{aligned} X_{0,\omega} &= \frac{4\pi}{a} ((1-\alpha)(1-d_1))^{i\eta_c} \left(\frac{(1-\beta)(b_1 + d_1 - 1)}{d_1 - 1} \right)^{i\eta_a} \left(\frac{(1-\gamma)(-c_1 - d_1 + 1)}{1-d_1} \right)^{i\eta_b} \times \\ &\times \left(\frac{(1-c_1)(b_1 + d_1 - 1)}{c_1 + d_1 - 1} \right)^{1+i\eta_c} \sum_{m=0}^{\infty} \frac{(-i\eta_a)_m}{m!} \left(\sum_{n=0}^m \frac{(-i\eta_c)_{m-n} (1+i\eta_c)_n}{(m-n)!n!} \left(\frac{b_1}{1-c_1} \right)^n \right) \times \end{aligned}$$

$$\begin{aligned}
& \times \left(\frac{b_1 c_1 + (d_1 - 1) r_1}{(c_1 + d_1 - 1)(b_1 + d_1 - 1)} \right)^{m-n} {}_2F_1 \left(1 + i\eta_a, n + 1 + i\eta_c; m - n + 1; \frac{b_1}{1 - d_1} \right) \times \\
& \times \sum_{k=0}^n \left(\frac{(-i\eta_b)_{m-n} (1 + i\eta_b)_k}{(k + m - n)!} \binom{n}{k} \left(\frac{c_1}{d_1 - 1} \right)^k \times \right. \\
& \left. \times {}_2F_1 \left(k + 1 + i\eta_b, n + 1 + i\eta_c; k + m - n + 1; \frac{c_1 d_1}{(1 - c_1)(1 - d_1)} \right) \right) \Bigg) \Bigg) . \tag{39}
\end{aligned}$$

Parameters b_1, c_1, d_1 are defined by the relations (27), and parameters α, β, γ are defined above as one of two solutions (26) of the system of quadratic equations (25). And, finally, the parameters included in right-hand side of the relations (27) and (26) are expressed through wave vectors by means of the relations (22). Here it should be noted that very ordinary power expansion of the function $(1 - \nu c_1 - u b_1)^{-1 - i\eta_c}$ gave rise to an appearance of the Appell function, not reducing to the Gauss hypergeometric function, and to an appearance of the additional limitations (30).

4. The simplest analytical representation of the overlap integral

Contrary to that, the application of the method of infinitesimal linear-fractional transformations to expand as well a power function $(1 - \nu c_1 - u b_1)^{-1 - i\eta_c}$ results in the final analysis in the Gauss hypergeometrical function instead of the Appell function in the terms of the two-fold series. We have showed that we obtain then, within denotations for intermediate parameters, the same representation for the overlap integral (17) as found in Ref. [1]. Such result, in actual fact, is to be expected. Indeed, as a result of applying of three linear-fractional transformations to the initial three-fold contour integral (21) and of obtaining of the internal integral by taking of a residue in a pole, we find the equivalent representation (29) of the overlap integral (17). But parameters, including expressions that depend on particles momenta are expressed through radicals in according with (26) and (27).

In spite of that, when we apply the method of infinitesimal linear-fractional transformations to expand both power functions in (29) in the process of transition to limits with respect to infinitesimal parameters all the above mentioned radicals disappear from limiting expressions. Then overlap integral (17) is presented in a form of the following two-fold series:

$$\begin{aligned}
X_{0,\omega} &= \frac{4\pi}{a-d} \left(\frac{a(a-b-d+g)(a-c-d+h)}{(a-b)(a-c)(a-d)} \right)^{1+i\eta_c} \left(1 - \frac{b-g}{a-d} \right)^{i\eta_a} \left(1 - \frac{c-h}{a-d} \right)^{i\eta_b} \times \\
& \times \sum_{m=0}^{\infty} \left(\frac{(-i\eta_a)_m (-i\eta_b)_m}{m! m!} \sum_{n=0}^m \frac{(1+i\eta_c)_n (-i\eta_c)_{m-n}}{n! (m-n)!} \left(\frac{bc-af}{(a-b)(a-c)} \right)^n \times \right. \\
& \times \left(\frac{f(d-a)+c(b-g)-h(b-g)}{(a-b-d+g)(a-c-d+h)} \right)^{m-n} {}_2F_1 \left(1 + i\eta_a, n + 1 + i\eta_c; m + 1; \frac{bd-ag}{(a-b)(a-d)} \right) \times \\
& \left. \times {}_2F_1 \left(1 + i\eta_b, n + 1 + i\eta_c; m + 1; \frac{cd-ah}{(a-c)(a-d)} \right) \right) \Bigg) . \tag{40}
\end{aligned}$$

As a result of comparison of the relations (22) with the similar relations (26) from Ref. [1] we find such substitution's relations:

$$a \rightarrow a_o, \quad b \rightarrow b_o, \quad c \rightarrow d_o, \quad f \rightarrow -h_o, \quad d \rightarrow a_o - c_o, \quad g \rightarrow b_o - f_o, \quad h \rightarrow d_o - g_o .$$

Executing these replacements, we will find algebraic representation of the our overlap integral in the following form:

$$\begin{aligned}
X_{0,\omega} &= \frac{4\pi}{c_o} \left(\frac{c_o - g_o}{c_o} \right)^{1+i\eta_c} \left(\frac{a_o(c_o - f_o)}{(a_o - b_o)(a_o - d_o)} \right)^{1+i\eta_c} \left(1 - \frac{f_o}{c_o} \right)^{i\eta_a} \left(1 - \frac{g_o}{c_o} \right)^{i\eta_b} \times \\
&\times \sum_{m=0}^{\infty} \frac{(-i\eta_a)_m (-i\eta_b)_m}{m! m!} \left(\sum_{n=0}^m \frac{(-i\eta_c)_{m-n} (1+i\eta_c)_n}{(m-n)! n!} \left(\frac{b_o d_o + a_o h_o}{(a_o - b_o)(a_o - d_o)} \right)^n \right) \times \\
&\times \left(\frac{f_o g_o + c_o h_o}{(c_o - f_o)(c_o - g_o)} \right)^{m-n} {}_2F_1 \left(1+i\eta_a, n+1+i\eta_c; m+1; \frac{a_o f_o - b_o c_o}{(a_o - b_o)c_o} \right) \times \\
&\times {}_2F_1 \left(1+i\eta_b, n+1+i\eta_c; m+1; \frac{a_o g_o - c_o d_o}{c_o(a_o - d_o)} \right).
\end{aligned} \tag{41}$$

We have derived exactly the same representation, that in Ref. [1], with the same definition of initial parameters, namely:

$$\begin{aligned}
a_o &\rightarrow q^2 + \omega^2, \quad b_o \rightarrow -2((\vec{q} \cdot \vec{k}_a) - i\omega k_a), \quad d_o \rightarrow -2((\vec{q} \cdot \vec{p}_b) - i\omega p_b), \\
c_o &\rightarrow q^2 + \omega^2 - 2((\vec{q} \cdot \vec{q}_c) + i\omega q_c), \quad f_o \rightarrow 2(-(\vec{q} \cdot \vec{k}_a) + i\omega k_a + (\vec{k}_a \cdot \vec{q}_c) + k_a q_c), \\
g_o &\rightarrow 2(-(\vec{q} \cdot \vec{p}_b) + i\omega p_b + (\vec{p}_b \cdot \vec{q}_c) + p_b q_c), \quad h_o \rightarrow 2(k_a p_b - (\vec{p}_b \cdot \vec{k}_a)).
\end{aligned} \tag{42}$$

Obviously, that an expression (41) is more simpler than even an expression (40).

5. Conclusions

The method of calculation of Coulomb desintegration of a projectile in an external region, offered in this work, uses the distorted waves approximation while the main part of a Coulomb breakup amplitude is determined in zero-range approximation for interaction of clusters composing a projectile. The effects of its structure can be taken into account by the corrections of the first and second orders due to a finite radius of clusters interaction. In the presented method these corrections are determined by expansion of wave functions of ejecting particles with respect to powers of relation (r/R) , which is assumed to be small in external region. Then a characteristic quantity which determines an exactness of calculation of a Coulomb breakup amplitude will be $(r_{\max}/R_{\max})^2$, where r_{\max} and R_{\max} set the radial sizes of the model space in the CDCC method. Then, for example, applying our method to the treatment of data [16] of the GSI experiment, for good channels discretizing it is possible to choose $r_{\max} = 200$ fm and $R_{\max} = 800$ fm. In this case $K_{\max} = 27.9$ fm⁻¹ and $L_{\max} = 22311$. Such an order of values of angular moments requires the special method of solving of the coupled equations system of discretized channels.

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ОБ АЛЬТЕРНАТИВНОМ ПОДХОДЕ К АНАЛИТИЧЕСКОМУ ПРЕДСТАВЛЕНИЮ АМПЛИТУДЫ КУЛОНОВСКОГО РАЗВАЛА

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Предложен обновленный метод учета вклада кулоновского взаимодействия от внешней пространственной области в амплитуду реакции развала, учитывающий поправки первого и второго порядков на конечность радиуса взаимодействия двух кластеров, которые образуют распадную налетающую частицу. Применение этого метода как составной части метода связанных каналов с дискретизацией континуума должно значительно сократить размеры требуемого модельного пространства и также число связанных уравнений, подлежащих решению. Показано, что подход, не использующий метод инфинитизимальных дробно-линейных преобразований для алгебраического представления главной части амплитуды кулоновского развала, в результате дает более сложные выражения, чем с применением этого метода. Таким образом, в результате двукратного применения указанного преобразования к интегралу для амплитуды кулоновского развала, определенной в приближении нулевого радиуса, получено алгебраическое выражение наиболее простого вида.

ПРО АЛЬТЕРНАТИВНИЙ ПІДХІД ДО АНАЛІТИЧНОГО ПРЕДСТАВЛЕННЯ АМПЛІТУДИ КУЛОНІВСЬКОГО РОЗВАЛУ

А. П. Ільїн

Запропоновано оновлений метод урахування вкладу кулонівської взаємодії від зовнішньої просторової області в амплітуду реакції розвалу, що враховує поправки першого та другого порядків на кінець радіуса взаємодії двох кластерів, які утворюють розпадну налітаючу частинку. Використання цього методу як складової частини методу зв'язаних каналів з дискретизацією континуума повинне значно скоротити розміри необхідного модельного простору і також число зв'язаних рівнянь, що підлягають розв'язку. Показано, що підхід, котрий не використовує метод інфінітізимальних дробово-лінійних перетворень для представлення в алгебраїчному виді головної частини амплітуди кулонівського розвалу, в результаті дає складніші вирази, ніж із застосуванням цього методу. Таким чином, у результаті двократного застосування вказаного перетворення до інтеграла для амплітуди кулонівського розвалу, визначеної в наближенні нульового радіуса, отримано алгебраїчний вираз, що має найпростіший вид.