

BUBBLE INSTABILITY OF HOT NUCLEAR MATTER

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Within the nuclear Fermi-liquid approach the bubble instability growth rate of a hot nuclear matter is considered. The influence of the damping and memory effects on the growing and collapsing bubbles is studied.

1. The equilibrium state of the nuclear matter (nuclear Fermi-liquid) is stable with respect to the particle density and Fermi-surface distortions. With decreasing bulk density or increasing temperature the nuclear matter reaches the regions of mechanical or thermodynamical instabilities with respect to small particle density distortions and to separation into liquid and gas phases. Moreover, under certain conditions (see below) the generation and the growth of the vapor phase (vapor bubbles) inside the liquid phase occur as a result of the heterophase fluctuations. This so-called bubble instability is responsible for the boiling process in the liquid and for the breakdown (multifragmentation) of the liquid at high temperatures. For a small embryonic bubble, the dynamics of the bubble instability is influenced significantly by the fluctuation of the particle number in the bubble region. This effect can be considered [1, 2] including a random force in the macroscopic equation of motion for the bubble radius. The important feature of the development of the instabilities in a Fermi-liquid is its hindrance by the dynamic Fermi surface effect [3]. In the present work, we will consider the dynamic evolution of the bubble in the nuclear Fermi liquid, taking into account the influence of the Fermi motion of nucleons in the surrounding Fermi liquid on the bubble dynamics.

Let us first consider some peculiarities of the development of the instabilities in a Fermi liquid. Considering the small particle density fluctuations $\delta\rho(\mathbf{r}, t)$, one can derive the linearized equation of motion for the Fermi liquid in the following form (see Ref. [4])

$$m \frac{\partial^2}{\partial t^2} \delta\rho = \bar{\nabla} \rho_{\text{eq}} \bar{\nabla} \frac{\delta\Phi}{\delta\rho} + \nabla_\nu \nabla_\mu P'_{\nu\mu}, \quad (1)$$

where ρ_{eq} is the equilibrium density, Φ is the thermodynamical potential and the pressure tensor, $P'_{\nu\mu}$, represents the deviation of the pressure from its isotropic part due to the Fermi surface distortions. The variational derivative $\delta\Phi/\delta\rho$ in Eq. (1) implies a linearization with respect to the density variation $\delta\rho$:

$$\frac{\delta\Phi}{\delta\rho} = \left(\frac{\delta\Phi}{\delta\rho} \right)_{\text{eq}} + \hat{L}[\rho_{\text{eq}}] \delta\rho + O(\delta\rho)^2. \quad (2)$$

The operator \hat{L} can be obtained from the equation of state $\Phi = \Phi[\rho]$ and it is given by [3]

$$\hat{L}[\rho_{\text{eq}}] \delta\rho = \frac{K}{9} \nabla^2 \delta\rho - 2(\beta + \alpha_s \rho_{\text{eq}}) \nabla^2 \nabla^2 \delta\rho, \quad (3)$$

where

$$\beta = \frac{\hbar^2}{18m}, \quad \alpha_s = \frac{1}{64}(9t_1 - 5t_2), \quad K = 6\varepsilon_F(1 + F_0) \left(1 + \frac{1}{3}F_1 \right)^{-1}, \quad (4)$$

t_k ($k = 0, 1, 2, 3$) is the parameter of Skyrme forces [5] and K is the incompressibility. Here, the Landau's amplitudes F_l are given by

$$F_0 = \frac{9\rho_{\text{eq}}}{8\varepsilon_F} \left[t_0 + \frac{3}{2}t_3 \rho_{\text{eq}} \right] \frac{m^*}{m} + 3 \left(1 - \frac{m^*}{m} \right), \quad F_1 = 3 \left(\frac{m^*}{m} - 1 \right), \quad (5)$$

where

$$\frac{m}{m^*} = 1 + \frac{m \rho_{\text{eq}}}{8 \hbar^2} (3t_1 + 5t_2)$$

and ε_F is the Fermi energy. The pressure tensor $P'_{\nu\mu} \equiv P'_{\nu\mu}(\mathbf{r}, t)$ in Eq. (1) is caused by the Fermi surface distortion. In the case of the most important quadrupole distortion of the Fermi surface, the pressure tensor $P'_{\nu\mu}$ satisfies the following equation [3]

$$\frac{\partial}{\partial t} P'_{\nu\mu} + P \frac{\partial}{\partial t} \Lambda_{\nu\mu} = -\frac{1}{\tau} P'_{\nu\mu} \quad \text{with} \quad \Lambda_{\nu\mu} = \nabla_\nu \chi_\mu + \nabla_\mu \chi_\nu - \frac{2}{3} \delta_{\nu\mu} \nabla_\lambda \chi_\lambda, \quad (6)$$

where τ is the relaxation time, $\vec{\chi}(\mathbf{r}, t)$ is the displacement field and $P \equiv P(\mathbf{r}, t) = (2/3)\varepsilon_{\text{kin}} \approx (3/5)\rho\varepsilon_F$ is the isotropic part of the pressure.

One can see from Eq. (4) that the Fermi liquid is unstable ($K < 0$) if $F_0 < -1$. In this case, the solution to Eq. (1) is given by a time increasing mode

$$\delta\rho \sim \exp(\Gamma t - i\mathbf{q} \cdot \mathbf{r}), \quad (7)$$

where $\Gamma > 0$ is the instability growth rate. The dynamics of the Fermi liquid is significantly influenced by the Fermi surface distortion (FSD) effects [3]. Due to this fact, the instability growth rate Γ of the Fermi liquid in the unstable region $F_0 < -1$ is strongly reduced, when $0 < -1 - F_0 \ll 1$. The FSD effects are negligible in the region when $F_0 \ll -1$. (Note that, in general, the limit $|F_0| \rightarrow \infty$ represent so-called hydrodynamic regime, where the dynamics of the Fermi liquid is similar to the dynamics of the classical liquid like a water.) Solving Eq. (1) for the hydrodynamic regime, we obtain the following result (see also Ref. [6])

$$\Gamma \Rightarrow \Gamma_1 \approx \frac{1}{\sqrt{3}} \frac{p_F}{m^*} q (|F_0| - 1)^{1/2}. \quad (8)$$

In a similar manner, for the Fermi liquid regime $|F_0| \rightarrow 1$ with $0 < -1 - F_0 \ll 1$, the result for the instability growth rate Γ reads

$$\Gamma \Rightarrow \Gamma_0 \approx \frac{2}{\pi} \frac{p_F}{m^*} q \frac{|F_0| - 1}{|F_0|}. \quad (9)$$

One can see from Eqs. (8) and (9) that the instability growth rate Γ of the Fermi liquid in the unstable region $F_0 < -1$ is strongly reduced, by the factor $\sim (|F_0| - 1)^{1/2}$ when $0 < -1 - F_0 \ll 1$.

The influence of the FSD effect on the development of instabilities in a Fermi liquid is the quite general phenomenon. Below, we will study it for the case of the bubble instability for both the boiling and the bubble collapse.

2. The necessary condition for the boiling is the equilibrium between the liquid and saturated vapor phases. However the boiling can only start in a metastable phase (overheated or extended liquid) [7]. In an one-component classical liquid, the dynamics of the vapor bubble of radius R is mainly derived by the thermodynamical potential $\Phi(R)$. The thermodynamic equilibrium between the vapor bubble and the surrounding metastable liquid is reached at the critical radius $R = R^*$ [7, 8] where the thermodynamical potential $\Phi(R)$ takes a *maximum* value and the critical radius R^* of the bubble in an overheated liquid at the temperature T is given by

$$R^* = \frac{2\sigma T_0}{\rho_{\text{vap}} _ \phi \Delta T}. \quad (10)$$

Here, T_0 is the boiling temperature in the case of plane geometry, σ is the surface tension coefficient, ρ_{vap} is the particle density in the bubble, $_ \phi$ is the latent heat of evaporation and $\Delta T = T - T_0$ is the overheating

temperature. The maximum (not minimum, as in the case of ordinary equilibrium) value of the thermodynamical potential $\Phi(R)$ at $R = R^*$ means that the vapor bubble is unstable with respect to a variation of radius R providing the boiling up at $R > R^*$. In a two-component nuclear liquid, the thermodynamical potential $\Phi(R, X_b)$ depends on both the bubble radius R and the isotopic asymmetry parameter X_b of the confined vapor. (Asymmetry parameter X_b is defined as $X_b = (N_b - Z_b)/(N_b + Z_b)$, where N_b and Z_b are the number of neutrons and protons in the bubble). The thermodynamic equilibrium is then reached at $R = R^*$ and $X_b = X_b^*$. In nuclear matter, the equilibrium asymmetry X_b^* of the vapor bubble exceeds the asymmetry of the surrounding liquid [8] and (R^*, X_b^*) defines the saddle point of two-dimension surface $\Phi = \Phi(R, X_b)$. Assuming $X_b = X_b^*$, the value of thermodynamical potential $\Phi(R)$ is given by

$$\Phi = \Phi_{\text{eq}} + \Delta\Phi, \quad \Delta\Phi \equiv \Delta\Phi(R) = 4\pi\sigma \left(R^2 - \frac{2R^3}{3R^*} \right). \quad (11)$$

The thermodynamical potential of Eq. (11) corresponds to the thermodynamically unstable (metastable) liquid phase with $\mu_{\text{liq}} > \mu_{\text{vap}}$, where μ_{liq} and μ_{vap} are the chemical potentials for the liquid and the bubble vapor, respectively.

We will apply the fluid dynamic equation of motion (1) to the bubble dynamics. In this case, considering the dynamics of the surrounding Fermi liquid, we have to include to Eq. (1) the additional external force \mathbf{F}_{ext} caused by the bubble vapor pressure on the liquid. Using the continuity equation for the velocity field $\mathbf{u} = \partial\chi/\partial t$ and Eq. (1), one obtains the Euler-like equation

$$m \frac{\partial}{\partial t} u_v + \nabla_v \frac{\delta\Phi}{\delta\rho} = -\frac{1}{\rho} \nabla_\mu P'_{v\mu} + F_{v,\text{ext}}. \quad (12)$$

The external force \mathbf{F}_{ext} on the bubble surface in the radially outward direction per unit area is given by [9, 10]

$$\bar{F}_{\text{ext}} = P_{\text{vap}} - P_0, \quad (13)$$

where P_{vap} is the vapor pressure within the bubble and P_0 is the pressure of the saturated vapor with respect to a plane surface. Note that the condition of equilibrium between the bubble vapor and the liquid is expressed by $P_0 = P_{\text{vap}}$ and $\bar{F}_{\text{ext},\text{eq}} = 0$, see Ref. [7].

The fluid dynamic equation of motion (12) can be reduced to the macroscopic one for the bubble radius $R(t)$. To achieve this goal, we point out that the continuity equation for the velocity field \mathbf{u} , taken on the surface of the spherical bubble, leads to the following form of the displacement field χ in the surrounding liquid [11]

$$\chi_v(r, t) = \frac{R^3}{3r^2} \frac{r_v}{r}, \quad r \geq R. \quad (14)$$

The displacement field of Eq. (14) allows us to evaluate the collective kinetic energy E_{kin} and the corresponding inertial parameter B . Namely,

$$E_{\text{kin}} = \frac{m}{2} \int d\mathbf{r} \rho u^2 = \frac{1}{2} B \dot{R}^2 \quad (15)$$

and, assuming $\rho_{\text{vap}} \ll \rho_{\text{liq}} \approx \rho = \rho_0 \theta(r - R)$, the inertial parameter is given by

$$B = 4\pi m \rho_0 R^3. \quad (16)$$

Multiplying Eq. (12) by $-\chi_\nu(\mathbf{r}, t) = R^2 r_\nu / r^3$, summing over ν , integrating over \mathbf{r} -space and using Eqs. (6), we obtain the non-Markovian equation for the variable $R(t)$

$$B\ddot{R} + \frac{1}{2} \frac{\partial B}{\partial R} \dot{R}^2 + I(R; t) = -\frac{\partial \Phi}{\partial R} - 4\pi R^2 P_0 (1 - P_{\text{vap}}/P_0), \quad (17)$$

where

$$I(R; t) = B \int_{t_0}^t dt' \dot{R}(t') \exp[(t' - t)/\tau] K(t, t') \quad (18)$$

is the memory integral

$$K(t, t') = \frac{8\varepsilon_F}{5mR(t)R(t')}. \quad (19)$$

The vapor pressure in the bubble, P_{vap} , includes both the quantum and the thermal contributions. Since the temperature region of interest is $T \ll \varepsilon_F$, the thermal pressure is relatively small and for the degenerate neutron gas one can use the Thomas-Fermi approximation with $P_{\text{vap}}/P_0 = (R^*/R)^5$ in Eq. (17).

We point out that the non-Markovian form of Eq. (17) is due to the effects of the Fermi surface distortion. The memory integral $I(R; t)$ of Eq. (18) provides both the friction and the conservative time-reversible force in Eq. (17). In both limits $\tau \rightarrow 0$ and $\tau \rightarrow \infty$, the macroscopic equation (17) is reduced to the usual Markovian (no memory) equation with a friction coefficient γ

$$m\rho_0 R\ddot{R} + \frac{3}{2} m\rho_0 \dot{R}^2 + \gamma \dot{R} = -\frac{2\sigma}{R} \left(1 - \frac{R}{R^*}\right) - P_0 \left(1 - \left(\frac{R^*}{R}\right)^5\right). \quad (20)$$

The friction coefficient γ is related to the relaxation time τ as [15]

$$\gamma = \omega_F B^* \frac{\omega_F \tau}{1 + (\omega_F \tau)^2}, \quad (21)$$

where $B^* \equiv B(R^*)$ is the mass coefficient from Eq. (16) at $R = R^*$, $\omega_F = \sqrt{\tilde{\kappa}/B^*}$ is the characteristic frequency for the eigenvibrations caused by the Fermi surface distortion and $\tilde{\kappa} = (32/5)\pi\rho_0\varepsilon_F R^*$ is the corresponding stiffness coefficient. Note that the equation of motion (20) represents the generalized Rayleigh-Plesset equation of classical bubble dynamics [10].

In the case of boiling of asymmetric nuclear matter, the vapor asymmetry X_{vap} significantly exceeds the corresponding liquid asymmetry X_{liq} (at $X_{\text{liq}} > 0$), see Ref. [12]. This is a feature of the nuclear matter where the structure of the isospin symmetry energy provides the condition $|\mu_n| < |\mu_p|$ and the preferable evaporation of neutrons. Using this fact, we will neglect the proton fraction in the bubble vapor. The presence of the noncondensable neutron vapor within the bubble then gives a contribution to the term $\sim (1 - P_{\text{vap}}/P_0)$ in Eq. (17). (Note that the analogous term in the classical Rayleigh-Plesset equation is due to the assumption that the bubble contains some quantity of contaminant noncondensable gas [10].)

3. We will consider the collapse phase, i.e. the descent from the top of the barrier of $\Phi(R)$, see Eq. (11), toward $R < R^*$. If the collapse process is fast enough and the condensation of the neutron's vapor within the bubble is negligible, the force \bar{F}_{ext} of Eq. (13) has then to be taken into consideration in Eq. (12). In an opposite case of a slow collapse process, the possible growth of the pressure P_{vap} at the decrease of the bubble radius $R(t)$ is compensated by the condensation of the vapor providing that $P_{\text{vap}} = P_0 = \text{const}$ and $\bar{F}_{\text{ext}} = 0$.

The non-Markovian equation (17) can be solved if one rewrites it as a set of two connected equations for the bubble radius $R(t)$ and the memory integral $I(R; t)$ with the relevant initial conditions. The bubble starts

to collapse from the metastable state at $R(0) = R^*$. To evaluate the critical radius R^* we point out that the caloric curve has the plateau region at the temperature T_0 [12]. Below we will adopt $T_0 = 4$ MeV and $X_{\text{liq}} = 0.1$. If one assumes the process of isobaric heating for the description of the plateau region in the caloric curve at $T_0 = 4$ MeV, the order of magnitude of the pressure P_0 should be $P_0 \approx 10^{-3}$ MeV/fm³ for this process. Under these conditions, one obtains from Eq. (10) $R^* = Q/\Delta T$ with $Q \approx 4.1$ MeV·fm for a symmetric nuclear matter. The parameter Q grows slightly with the asymmetry of the nuclear matter. Taking the overheating temperature $\Delta T = 0.5$ MeV, we obtain from Eq. (10) $R(0) = R^* = 8.2$ fm. The initial velocity $\dot{R}(0)$ can be derived using the initial kinetic energy $E_{\text{kin},0}$. Assuming the equipartition of energy over degrees of freedom at $R = R^*$, we use $E_{\text{kin},0} = (T_0 + \Delta T)/2$ and obtain $\dot{R}(0) = -\sqrt{2E_{\text{kin},0}/B(R^*)} = -2.9 \cdot 10^{-3} c$, where sign “-” is used because of the collapse process with $R(t) < R^*$. The surface tension coefficient σ in Eq. (11) is temperature dependent. We will use the following expression for $\sigma \equiv \sigma(T)$ [13]

$$\sigma(T) = \sigma(0) \left[(T_{\text{crit}}^2 - T^2) / (T_{\text{crit}}^2 + T^2) \right]^{5/4}, \quad (22)$$

where $T_{\text{crit}} = 14.6$ MeV is the critical temperature for infinite nuclear Fermi-liquid, associated with the SKM interaction [12].

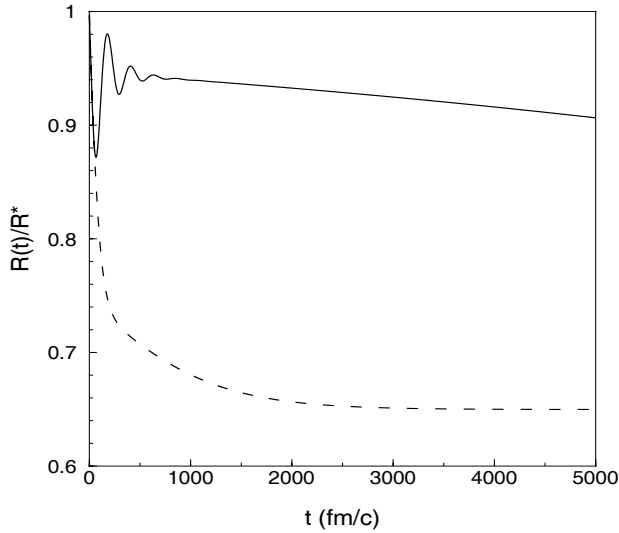


Fig. 1. Dependence of the radius $R(t)$ of the collapsing bubble on the time obtained for two regimes: non-Markovian motion, Eq. (17), (solid line) and Markovian (no memory) motion, Eq. (20), with the friction coefficient γ from Eq. (21) (dashed line). We have used the values of the initial radius $R(0) = R^* = 8.2$ fm, the initial velocity $\dot{R}(0) = -2.9 \cdot 10^{-3} c$ (see text).

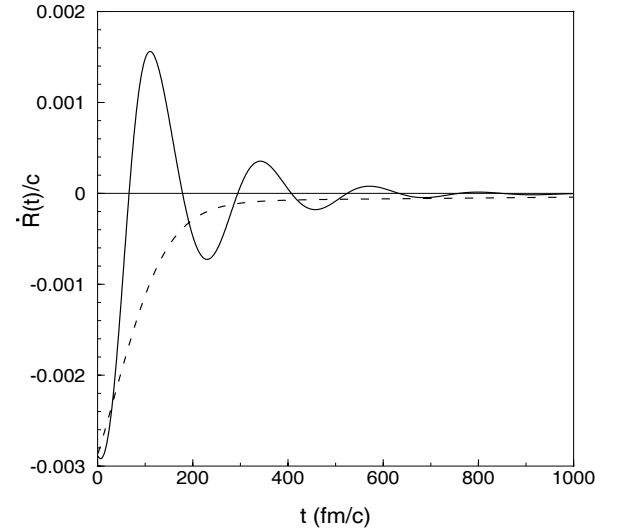


Fig. 2. The same as in Fig. 1 but for the velocity $\dot{R}(t) = dR(t)/dt$ (in the units of the light velocity c).

In Fig. 1, we show the time dependence of the radius $R(t)$ in the presence of the noncondensable vapor of neutrons with $\bar{F}_{\text{ext}} \neq 0$ (solid lines). Here we have adopted $\varepsilon_F = 37$ MeV, $\sigma(0) = 1.1$ MeV·fm⁻² and $\rho_0 = 0.16$ fm⁻³. For the relaxation time τ we have used $\tau = \tau_0/T^2$ with $\tau_0 = 850$ fm/c [14]. The dashed line in Fig. 1 shows the corresponding result for the Markovian (no memory) motion, Eq. (20) with the friction coefficient γ from Eq. (21). As seen from Fig. 1, the collapse of the bubble is accompanied by oscillatory behavior of its radius $R(t)$. These oscillations occur due to the memory integral in Eq. (17) and disappear in the limit of Markovian motion (see dashed line in Fig. 1). As seen from Fig. 1, the collapse process stops at a

certain values of the bubble radius, $R = R_{\min}$. The presence of non-zero $R_{\min} \neq 0$ is due to the pressure of the noncondensable neutron vapor in the bubble. The value of R_{\min} is derived by the condition of the compensation of the adiabatic forces in Eq. (17) at $t \gg \tau$. In Fig. 2 we show the velocity of the collapsing. As seen from Figs. 1 and 2 the presence of memory effect (solid line) strongly hinders the collapse process.

If the collapse process becomes slow enough the growth of the vapor pressure P_{vap} is compensated by the condensation of the vapor and the contribution from \bar{F}_{ext} is negligible in Eq. (17). The collapse process then leads to the disappearance of the bubble. The time, t_{clps} , required for the total collapse from $R = R^*$ to $R = 0$ is changed significantly due to the memory effects [16]. A delay in the collapse of the bubble occurs due the non-Markovian (memory) effect for the large values of the relaxation time. In the limit of a non-viscous liquid, $\tau \rightarrow 0$, we obtain $t_{\text{clps},0} = 4 \cdot 10^{-22}$ s. This result can be compared with the classical one of Rayleigh $t_{\text{clps,R}}$ [17]. To do that, we note that under the condition used above for $T_0 = 4$ MeV, $P_0 \approx 10^{-3}$ MeV/fm³, $X_{\text{liq}} = 0.1$ and $R^* = 8.2$ fm, the Fermi liquid is undercompressed by a pressure $\Delta P \approx 0.25$ MeV/fm³. The Rayleigh collapsing time $t_{\text{clps,R}}$ is given by [10]

$$t_{\text{clps,R}} = 0.915 \left(\frac{m \rho_{\text{liq}} R^2(0)}{\Delta P} \right)^{1/2}$$

with the yield $t_{\text{clps,R}} = 6.1 \cdot 10^{-22}$ s which is in a good agreement with $t_{\text{clps},0}$.

In conclusion we note that the growing and the collapse of the bubble in an overheated (undercompressed) Fermi liquid is strongly influenced by memory effects, if the relaxation time τ is large enough at $\omega_F \tau \gg 1$. In this case, the development of the bubble instability is accompanied by characteristic shape oscillations of the bubble (see Figs. 1 and 2) which depend on the memory kernel $K(t, t')$ and the relaxation time τ . These oscillations appear due to the non-adiabatic elastic force induced by the memory integral. The non-adiabatic elastic force acts against the adiabatic force $-\partial\Phi/\partial R$ (see Eq. (17)) and hinders the instability growth rate of the bubble. The delay in the growing and collapse of the bubble is caused here by both conservative elastic and friction forces. We should like to point out that the study of the cavitation in a two-component nuclear matter could represent a stimulating area of interest due to the fact that the vapor inside embryonic bubbles is strongly asymmetric system with $(N - Z)/(N + Z) \approx 1$, where N and Z are the number of neutrons and protons, respectively. The collapse of such extremely neutron rich system in which a huge amount of energy ($\sim 10^3$ MeV) is focused in a small region could show up new phenomena in the clusterization of nucleons.

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ПУЗЫРЬКОВАЯ НЕУСТОЙЧИВОСТЬ ГОРЯЧЕЙ ЯДЕРНОЙ МАТЕРИИ

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В рамках теории ядерной ферми-жидкости исследуется нарастание пузырьковой неустойчивости в горячей ядерной материи. Изучается влияние эффектов затухания и памяти на рост и коллапс пузырьков.

БУЛЬБАШКОВА НЕСТАБІЛЬНІСТЬ ГАРЯЧОЇ ЯДЕРНОЇ МАТЕРІЇ

В. М. Коломієць

У рамках теорії ядерної фермі-рідини досліджується наростання бульбашкової нестабільності в гарячій ядерній матерії. Вивчається вплив ефектів затухання та пам'яті на ріст та колапс бульбашок.