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MESON-TWO-NUCLEON CLOTHED GENERATORS OF THE POINCARÉ GROUP DERIVED FROM QUANTUM FIELD THEORY: MASS AND VERTEX RENORMALIZATION

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The method of unitary clothing transformation is used to deal with the so-called clothed particle representation in which generators of the Poincaré group acquire one and the same sparse structure in the Fock space of particle states. In the model of three-linear Yukawa type pseudoscalar interaction between nucleon and pion fields we calculate the mass and vertex corrections in the second and third orders in the coupling constant respectively. Being expressed through Lorentz-covariant quantities, the respective corrections are momentum independent and coincide with results obtained within Dyson-Feynman technique.

1. Introduction. Generators of the Poincaré group

Poincaré invariance requires that there exists a unitary representation of the Poincaré group defined in a Hilbert space. The corresponding ten generators fulfill the following set of commutation relations

$$[N_i, N_k] = -i\varepsilon_{ikl} J_l\,,\ [J_i, N_k] = i\varepsilon_{ikl} N_l\,,\ [J_i, J_k] = i\varepsilon_{ikl} J_l\,,$$

$$[H, N_l] = -iP_l\,,\ [P_i, N_l] = -i\delta_{il} H\,,\ [J_i, P_k] = i\varepsilon_{ikl} J_l\,,$$

$$[P_i, H] = 0\,,\ [J_i, H] = 0\,,\ [P_i, P_k] = 0\,,\ (i,k,l = 1,2,3)\,. \tag{1}$$

Here H is the Hamiltonian operator, P_i, J_i and N_i are the three components of the momentum, angular momentum and boost operators, respectively.

We note that in the instant form of relativistic dynamics after Dirac the four operators H and N_i carry interaction. It is well known that the one-particle eigenstates of H and N_i differ from the corresponding eigenstates of their free parts

$$H(\alpha)\left|\alpha^{\dagger}\Omega\right\rangle \neq H_F(\alpha)\left|\alpha^{\dagger}\Omega\right\rangle,$$

$$\boldsymbol{N}(\alpha)\left|\alpha^{\dagger}\Omega\right\rangle \neq \boldsymbol{N}_F(\alpha)\left|\alpha^{\dagger}\Omega\right\rangle. \tag{2}$$

Here via α we denote the whole set of creation and destruction operators of bare particles with bare masses which interact by means of bare coupling.

Therefore, one may ask a question whether there exists such a set of creation and destruction operators α_c for which the total Hamiltonin $K(\alpha_c)$, the boost operators $\boldsymbol{B}(\alpha_c)$ and their free counterparts $K_F(\alpha_c)$ and $\boldsymbol{B}_F(\alpha_c)$ would fulfill the conditions

$$K(\alpha_c)\left|\alpha_c{}^{\dagger}\Omega\right\rangle = K_F(\alpha_c)\left|\alpha_c{}^{\dagger}\Omega\right\rangle,$$

$$\boldsymbol{B}(\alpha_c)\left|\alpha_c{}^{\dagger}\Omega\right\rangle = \boldsymbol{B}_F(\alpha_c)\left|\alpha_c{}^{\dagger}\Omega\right\rangle, \tag{3}$$

without destroying the algebra?

Greenberg and Schweber [1] assumed that such set of operators α_c existed and was connected with the initial set of operators α via the unitary transformation which kept *S*-operator intact. The total Hamiltonian and the boost operators in terms of clothed particles can be written as

$$H(\alpha) = W(\alpha_c)H(\alpha_c)W^{\dagger}(\alpha_c) \equiv K(\alpha_c) = K_F(\alpha_c) + K_I(\alpha_c)\,, \tag{4}$$

$$\boldsymbol{N}(\alpha) = W(\alpha_c)\boldsymbol{N}(\alpha_c)W^{\dagger}(\alpha_c) \equiv \boldsymbol{B}(\alpha_c) = \boldsymbol{B}_F(\alpha_c) + \boldsymbol{B}_I(\alpha_c)\,, \tag{5}$$

$$W = W(\alpha) = W(\alpha_c) = \exp R(\alpha_c), \tag{6}$$

$$\alpha_c = W^\dagger \alpha W, \; WW^\dagger = W^\dagger W = 1, \tag{7}$$

$$\alpha = W(\alpha_c)\alpha_c W^\dagger(\alpha_c). \tag{8}$$

Shebeko and Shirokov have shown [2] that the generator R of the unitary clothing transformation W can be chosen in such a way that the clothed vacuum and all the one-clothed-particle states are simultaneously the eigenstates of $K(\alpha_c)$, $\boldsymbol{B}(\alpha_c)$ and their free parts up to the second order in the coupling constant.

2. Clothed particles in quantum field theory

The notion of clothed particles will be considered using the field model in which a spinor fermion (nucleon) field interacts with a neutral meson (pion) field by means of Yukawa-type three linear interaction with pseudoscalar (PS) coupling. The initial Hamiltonian has the form

$$H = H(\alpha) = H_F(\alpha) + H_I(\alpha), \tag{9}$$

where $H_F(\alpha)$ is the free part

$$H_F = \int d\mathbf{k}\omega_{\mathbf{k}} a^\dagger(\mathbf{k})a(\mathbf{k}) + \int d\mathbf{p}E_{\mathbf{p}}\sum_r \left[b^\dagger(\mathbf{p},r)b(\mathbf{p},r) + d^\dagger(\mathbf{p},r)d(\mathbf{p},r)\right], \tag{10}$$

and $H_I(\alpha)$ is the interaction one

$$H_I(\alpha) = V(\alpha) + M_{ren,mes}(\alpha) + M_{ren,\,ferm}(\alpha) + V_{ren}(\alpha). \tag{11}$$

In these formulae $E_{\mathbf{p}} = \sqrt{\mathbf{p}^2 + m^2}$ and $\omega_{\mathbf{k}} = \sqrt{\mathbf{k}^2 + \mu^2}$, m and μ are the nucleon and meson physical masses; $\mathbf{p}$ and $\mathbf{k}$ are the nucleon and meson momenta; r is the spin projection index. The creation (destruction) operators of mesons $a^\dagger(\mathbf{k})\left(a(\mathbf{k})\right)$ and the same for fermions $b^\dagger(\mathbf{p},r)\left(b(\mathbf{p},r)\right)$ and antifermions $d^\dagger(\mathbf{p},r)\left(d(\mathbf{p},r)\right)$ satisfy the commutation relations

$$\left[a(\mathbf{k}), a^\dagger(\mathbf{k}')\right] = \delta(\mathbf{k}-\mathbf{k}'), \left\{b(\mathbf{p},r), b^\dagger(\mathbf{p}',r')\right\} = \left\{d(\mathbf{p},r), d^\dagger(\mathbf{p}',r')\right\} = \delta_{r,r'}\delta(\mathbf{p}-\mathbf{p}'). \tag{12}$$

The interaction operator $V(\alpha)$ has the following form

$$V(\alpha) = \int d\mathbf{k}\hat{V}^{\mathbf{k}} a(\mathbf{k}) + H.c., \tag{13}$$

$$\hat{V}^{\mathbf{k}} = \int d\mathbf{p}' d\mathbf{p} \sum_{r,r'}\sum_{i,j} F_i^\dagger(\mathbf{p}',r') V_{i,j}^{\mathbf{k}}(\mathbf{p}',r';\mathbf{p},r) F_j(\mathbf{p},r), \tag{14}$$

$$V_{i,j}^{\mathbf{k}}(\mathbf{p}',r';\mathbf{p},r) = \frac{ig}{(2\pi)^{3/2}} \frac{m}{\sqrt{2\omega_{\mathbf{k}}E_{\mathbf{p}'}E_{\mathbf{p}}}} \delta(\mathbf{p}+\mathbf{k}-\mathbf{p}') \bar{U}_i(\mathbf{p}',r')\gamma_5 U_j(\mathbf{p},r). \tag{15}$$

Here we have introduced the following notations

$$U(\mathbf{p},r) = \begin{pmatrix} U_1(\mathbf{p},r) \\ U_2(\mathbf{p},r) \end{pmatrix} = \begin{pmatrix} u(\mathbf{p},r) \\ v(-\mathbf{p},r) \end{pmatrix}, \; F(\mathbf{p},r) = \begin{pmatrix} F_1(\mathbf{p},r) \\ F_2(\mathbf{p},r) \end{pmatrix} = \begin{pmatrix} b(\mathbf{p},r) \\ d^\dagger(-\mathbf{p},r) \end{pmatrix}, \tag{16}$$

where $u(\mathbf{p},r)$ and $v(\mathbf{p},r)$ are the spinors which satisfy the usual Dirac equations with physical masses and g is the physical coupling constant.

The usual meson and fermion mass counterterms $M_{ren,mes}(\alpha)$ and $M_{ren,ferm}(\alpha)$ can be presented as

$$M_{ren,mes}(\alpha)=\frac{\mu_0^2-\mu^2}{4}\int\frac{d\mathbf{k}}{\omega_{\mathbf{k}}}\left(a^\dagger(\mathbf{k})a(\mathbf{k})+a(\mathbf{k})a(-\mathbf{k})+H.c.\right), \tag{17}$$

$$M_{ren,ferm}(\alpha)=m\left(m_0-m\right)\int\frac{d\mathbf{p}}{E_{\mathbf{p}}}\sum_{r,r'}\sum_{i,j}F_i^\dagger\left(\mathbf{p},r'\right)M_{i,j}(\mathbf{p},r';\mathbf{p},r)F_j\left(\mathbf{p},r\right), \tag{18}$$

with $M_{i,j}(\mathbf{p},r';\mathbf{p},r)=\bar{U}_i\left(\mathbf{p},r'\right)U_j\left(\mathbf{p},r\right)$, and m_0 and μ_0 denoting the nucleon and meson bare masses.

The vertex counterterm $V_{ren}(\alpha)$ is determined by the same formulae (13) - (15) as for $V(\alpha)$ but with g substituted by $\delta g\equiv g_0-g$ where g_0 refers to the bare coupling constant.

Bearing in mind that H is a polinomial in α and using Eq. (8), one can replace the bare operators by the clothed ones

$$H=H(\alpha)=H\left(W\left(\alpha_c\right)\alpha_c W^\dagger\left(\alpha_c\right)\right)=W\left(\alpha_c\right)H\left(\alpha_c\right)W^\dagger\left(\alpha_c\right)=$$

$$=K\left(\alpha_c\right)=H_F\left(\alpha_c\right)+V(\alpha_c)+M_{ren,mes}(\alpha_c)+M_{ren,ferm}(\alpha_c)+V_{ren}(\alpha_c)+\left[R,H_F\right]+\left[R,H_I\right]+$$

$$+\frac{1}{2}\left[R,\left[R,H_F\right]\right]+\frac{1}{2}\left[R,\left[R,H_I\right]\right]+\ldots \tag{19}$$

The operator $K\left(\alpha_c\right)$ represents the same Hamiltonian as $H(\alpha)$ but with another dependence on its argument.

Eq. (19) contains some undesirable terms of the g^1 and higher orders, called "bad" after [2], that prevent the no-clothed-particle state and one-clothed-particle states to be the eigenstates of the total Hamiltonian.

Let us eliminate from $K\left(\alpha_c\right)$ the bad terms of the g^1-order comprising the operator V. For this purpose we choose such R that

$$V+\left[R,H_F\right]=0\,. \tag{20}$$

Then Eq. (19) can be rewritten as

$$K\left(\alpha_c\right)=H_F\left(\alpha_c\right)+M_{ren}\left(\alpha_c\right)+V_{ren}\left(\alpha_c\right)+\frac{1}{2}\left[R,V\right]+\left[R,M_{ren}\right]+\frac{1}{3}\left[R,\left[R,V\right]\right]+\ldots. \tag{21}$$

In the model under consideration, the solution of Eq. (20) has the form [2]

$$R=\int d\mathbf{k}\hat{R}_c^{\mathbf{k}}a_c(\mathbf{k})-H.c.,$$

$$\hat{R}_c^{\mathbf{k}}=\int d\mathbf{p}'d\mathbf{p}\sum_{r,r'}\sum_{i,j}F_{c,i}^\dagger\left(\mathbf{p}',r'\right)R_{i,j}^{\mathbf{k}}\left(\mathbf{p}',r';\mathbf{p},r\right)F_{c,j}\left(\mathbf{p},r\right). \tag{22}$$

Here the operator column $F_c\left(\mathbf{p},r\right)$ is composed of the clothed operators $b_c\left(\mathbf{p},r\right)$ and $d_c^{\dagger}\left(-\mathbf{p},r\right)$. The explicit expression for the c-number matrix $R_{i,j}^{\mathbf{k}}$ can be presented as

$$R_{i,j}^{\mathbf{k}}\left(\mathbf{p}',r';\mathbf{p},r\right)=V_{i,j}^{\mathbf{k}}\left(\mathbf{p}',r';\mathbf{p},r\right)/\left[\left(-1\right)^{i-1}E_{\mathbf{p}'}-\left(-1\right)^{j-1}E_{\mathbf{p}}-\omega_{\mathbf{k}}\right],\ (i,j=1,2). \tag{23}$$

Having defined the operator R, we have removed from $K\left(\alpha_c\right)$ all the bad terms of the g^1- order. Howerer, r.h.s. of Eq. (21) contains bad terms of the g^2 and higher orders. Therefore, we need to remove bad terms from $K\left(\alpha_c\right)$ order by order.

3. Mass renormalization

The operator $K(\alpha_c)$ contains terms $M_{ren}(\alpha_c)$ and $[R,V]$ which are of the g^2-order. The commutator $[R,V]$ can be given as

$$[R,V] = \int d\mathbf{k}_1 d\mathbf{k}_2 \left[\hat{R}^{\mathbf{k}_1}, \hat{V}^{\mathbf{k}_2}\right] a_c(\mathbf{k}_1) a_c(\mathbf{k}_2)$$

$$+\int d\mathbf{k}_1 d\mathbf{k}_2 \left[\hat{R}^{\mathbf{k}_1}, \hat{V}^{-\mathbf{k}_2}\right] a_c^\dagger(\mathbf{k}_2) a_c(\mathbf{k}_1) + \int d\mathbf{k} \hat{R}^{\mathbf{k}} \cdot \hat{V}^{-\mathbf{k}} + H.c. \tag{24}$$

After normal ordering, it involves terms bilinear in the meson and fermion operators that can be cancelled by the respective counterparts from the operator $M_{ren}(\alpha_c) = M_{ren,mes}(\alpha_c) + M_{ren,ferm}(\alpha_c)$.

In the case of the meson mass shift of the g^2-order the corresponding correction [2]

$$\delta\mu^2 \equiv \mu_0^2 - \mu^2 = \frac{2g^2}{(2\pi)^3} \int \frac{d\mathbf{p}}{E_{\mathbf{p}}} \left\{ 1 + \frac{\mu^4}{4(pk)^2 - \mu^4} \right\}, \tag{25}$$

is independent of the meson momentum $\mathbf{k}$. Here we have introduced the 4-vectors $p = (E_{\mathbf{p}}, \mathbf{p})$ and $k = (\omega_{\mathbf{k}}, \mathbf{k})$. The same result can be obtained using the well-known Dyson-Feynman method.

In a similar way we can calculate the nucleon mass shift of the g^2-order [3]:

$$\delta m \equiv m_0 - m = \frac{g^2}{4m(2\pi)^3} \left[I_1(p) + I_2(p) \right],$$

$$I_1(p) = \int \frac{d\mathbf{k}}{\omega_{\mathbf{k}}} pk \left\{ \frac{1}{\mu^2 - 2pk} - \frac{1}{\mu^2 + 2pk} \right\},$$

$$I_2(p) = \int \frac{d\mathbf{q}}{E_{\mathbf{q}}} \left\{ \frac{m^2 - pq}{2\left[m^2 - pq\right] - \mu^2} + \frac{m^2 + pq}{2\left[m^2 + pq\right] - \mu^2} \right\}. \tag{26}$$

Here we have adopted the four-vector notation $q = (E_{\mathbf{q}}, \mathbf{q})$. In Eq. (26) all quantities, being expressed through the explicitly covariant integrals, are independent of the particle momenta and coincide with these ones obtained in Dyson-Feynman method.

4. Vertex renormalization

The operator $V_{ren}(\alpha_c) + [R, M_{ren}] + \frac{1}{3}[R,[R,V]]$ contains terms of the g^3-order, which replicate the operator structure of the interaction operator V and, thus, start the program of vertex renormalization. The commutator $[R, M_{ren}]$ has the form

$$[R, M_{ren}] = \left[R, M_{ren,mes}\right] + \left[R, M_{ren,ferm}\right], \tag{27}$$

$$\left[R, M_{ren,mes}\right] = \frac{\delta\mu^2}{2} \int \frac{d\mathbf{k}}{\omega_{\mathbf{k}}} \left(\hat{R}^{\mathbf{k}} + \hat{R}^{-\mathbf{k}\dagger} \right) a_c(\mathbf{k}) + H.c., \tag{28}$$

$$\left[R, M_{ren,\,ferm}\right] = m\delta m \int d\mathbf{k}\, \frac{ig}{(2\pi)^{3/2}} \frac{m}{\sqrt{2\omega_{\mathbf{k}} E_{\mathbf{p}'} E_{\mathbf{p}}}} \delta(\mathbf{p}+\mathbf{k}-\mathbf{p}') \times$$

$$\times F^{\dagger}_{c,i}(\mathbf{p}', r') \left\{ \bar{U}_i(\mathbf{p}', r') \gamma_5 \cdot \begin{bmatrix} \left(\hat{p}' - \hat{k} - m\right)^{-1}, i=1 \\ -\left(\hat{p}'_{-} + \hat{k} + m\right)^{-1}, i=2 \end{bmatrix} \cdot U(\mathbf{p}, r) + \right.$$

$$\left. + \bar{U}_i(\mathbf{p}', r') \cdot \begin{bmatrix} \left(\hat{p} + \hat{k} - m\right)^{-1}, j=1 \\ -\left(\hat{p}_{-} - \hat{k} + m\right)^{-1}, j=2 \end{bmatrix} \cdot \gamma_5 U(\mathbf{p}, r) \right\} F_{c,j}(\mathbf{p}, r)\, a_c(\mathbf{k}) + H.c.. \tag{29}$$

Here $p\_ = (E_{\mathbf{p}}, -\mathbf{p})$, $\hat{p} = p^{\nu}\gamma_{\nu}$ and γ^{ν} is the conventional Dirac matrices.

The commutator $\left[R,[R,V]\right]$ can be written as

$$\left[R,[R,V]\right] = \int d\mathbf{k}_1 d\mathbf{k}_2 d\mathbf{k}_3 \left\{ A_1(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3) a^{\dagger}(\mathbf{k}_2) a^{\dagger}(\mathbf{k}_1) a(\mathbf{k}_3) + \right.$$

$$+ A_2(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3) a^{\dagger}(\mathbf{k}_2) a(\mathbf{k}_1) a(\mathbf{k}_3) + A_3(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3) a(\mathbf{k}_2) a(\mathbf{k}_1) a(\mathbf{k}_3) +$$

$$\left. + A_4(\mathbf{k}_1,\mathbf{k}_2) a(\mathbf{k}_1) \delta(\mathbf{k}_2 - \mathbf{k}_3) \right\} + H.c.,$$

$$A_1(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3) = \left[\hat{R}^{\mathbf{k}_3}, \left[\hat{R}^{\mathbf{k}_1}, \hat{V}^{\mathbf{k}_2}\right]^{\dagger}\right], A_2(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3) = \left[\hat{R}^{\mathbf{k}_3}, \left[\hat{R}^{\mathbf{k}_1}, \hat{V}^{\mathbf{k}_2\dagger}\right] + \left[\hat{R}^{\mathbf{k}_2}, \hat{V}^{\mathbf{k}_1\dagger}\right]^{\dagger}\right],$$

$$A_3(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3) = \left[\hat{R}^{\mathbf{k}_3}, \left[\hat{R}^{\mathbf{k}_1}, \hat{V}^{\mathbf{k}_2}\right]\right],$$

$$A_4(\mathbf{k}_1,\mathbf{k}_2) = \left[\hat{R}^{\mathbf{k}_1}, \hat{R}^{\mathbf{k}_2}\right]\hat{V}^{\mathbf{k}_2\dagger} + 2\hat{R}^{\mathbf{k}_2}\left[\hat{R}^{\mathbf{k}_1}, \hat{V}^{\mathbf{k}_2\dagger}\right] + \hat{R}^{\mathbf{k}_2}\left[\hat{R}^{\mathbf{k}_2}, \hat{V}^{\mathbf{k}_1\dagger}\right]^{\dagger} +$$

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$$+2\left[\hat{R}^{\mathbf{k}_1}, \hat{V}^{\mathbf{k}_2}\right]\hat{R}^{\mathbf{k}_2\dagger} + \hat{V}^{\mathbf{k}_2}\left[\hat{R}^{\mathbf{k}_1}, \hat{R}^{\mathbf{k}_2\dagger}\right] + \left[\hat{R}^{\mathbf{k}_2}, \hat{V}^{\mathbf{k}_1}\right]\hat{R}^{\mathbf{k}_2\dagger}. \tag{30}$$

After normal ordering of fermionic operators in Eq. (30), several part of the commutator $\left[R,[R,V]\right]$ is cancelled with the corresponding part of the commutator $[R, M_{ren}]$, providing the meson and nucleon wave function renormalization in the lowest order in g. At the same time, another part of the commutator $\left[R,[R,V]\right]$ is cancelled with the respective part of V_{ren}, determining the charge shift in the g^3-order

$$\delta g = \delta g(\text{Feynman}-\text{like}) + \delta g(\text{off}-\text{energy}-\text{shell}), \tag{31}$$

$$\delta g(\text{Feynman}-\text{like}) = \frac{g^3}{(2\pi)^3 m^2}\left(\int \frac{d\mathbf{q}'}{\omega_{\mathbf{q}'}} \frac{p'q'}{\mu^2 - 2p'q} \frac{m^2 + p'p - p'q'}{\mu^2 - 2pq} + \right.$$

$$\left. + \int \frac{d\mathbf{q}}{E_{\mathbf{q}}} \frac{m^2 - p'q}{2\left[m^2 + pq\right] - \mu^2} \frac{m^2 + p'q - p'k}{\mu^2 - 2kq} + \int \frac{d\mathbf{q}}{E_{\mathbf{q}}} \frac{m^2 + p'q}{2\left[m^2 + p'q\right] - \mu^2} \frac{m^2 - p'q - p'k}{\mu^2 + 2kq} \right), \tag{32}$$

where $E_{\mathbf{q}} = \sqrt{\mathbf{q}^2 + m^2}$, $\omega_{\mathbf{q}} = \sqrt{\mathbf{q}^2 + \mu^2}$, $p' = (E_{\mathbf{p}'}, \mathbf{p}')$, $q' = (\omega_{\mathbf{q}}, \mathbf{q}')$. The momentum conservation law for the given vertex has the form $\mathbf{p} + \mathbf{k} = \mathbf{p}'$, where $\mathbf{p}$, $\mathbf{k}$ and $\mathbf{p}'$ are the nucleon and meson momenta before the interaction and the nucleon momentum after the interaction respectively.

In common practice, the charge shift is usually determined by subtracting two certain quantities calculated for some physical process, one of which having one of its vertices renormalized to some order in g. In the case of the covariant (on-energy-shell) Dyson-Feynman approach, these quantities are the scattering amplitudes. Being on the energy shell by the origin, their difference is always the on-energy-shell quantity, providing the respective charge shift only on the energy shell. To the contrary, in the unitary clothing transformation approach promoted here, the quantities under consideration are the respective interaction parts of the total Hamiltonian operator. Those operators of the physical interactions are always off the energy shell, yielding the correspondent charge shift being the off-energy-shell quantity. We have calculated the difference between the respective operators for the $\pi N \to \pi N$ interaction in the g^2 and g^4 orders and found the expression for the g^3 charge shift coinciding with the formulae (31) and (32). However, if from the very beginning the operators under consideration are projected on the energy shell the obtained expression for the charge shift coincides with the formula (32). That is why we divide the expression for δg (31) obtained via the cancellation of the vertex counterterm into the Feynman-like part (32) which is momentum independent and the off-energy-shell part which goes to zero on the energy shell. Note that Eq. (32) gives another representation for the well-khown respective Dyson- Feynman covariant integral.

4. Conclusion

We have shown how the mass and vertex renormalization is performed in the model of three-linear Yukawa-type PS interaction between the nucleon and neutral pion fields with help of the clothed particle representation in which the total Hamiltonian and the other generators of the Poincaré group acquire one and the same sparse structure in the Fock space of particle states. The respective mass and vertex counterterms are cancelled, at least partly, due to normal ordering of the clothed creation (destruction) operators involved in the commutators $[R,V]$ and $[R,[R,V]]$ respectively. Being expressed through Lorentz-covariant quantities, the respective corrections are momentum independent and coincide with those ones obtained within Dyson-Feynman method. At the same time, our detailed investigation shows that the unitary clothing transformation method eliminates bad terms simultaneously from both the total Hamiltonian and the boost generators, and, along with that, the corresponding commutation relations of the Poincaré algebra are fulfilled in the corresponding orders in the coupling constant.

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МЕЗОН-ДВУХНУКЛОННЫЕ ОДЕТЫЕ ГЕНЕРАТОРЫ ГРУППЫ ПУАНКАРЕ В КВАНТОВОЙ ТЕОРИИ ПОЛЯ: ПЕРЕНОРМИРОВКА МАССЫ И ЗАРЯДА

В. Ю. Корда, П. А. Фролов

Методом унитарных одевающих преобразований реализовано представление ”одетых” частиц, в котором генераторы группы Пуанкаре приобретают одну и ту же разреженную структуру в фоковском пространстве состояний частиц. В модели взаимодействующих посредством трилинейной псевдоскалярной связи типа Юкавы нуклонного и нейтрального пионного полей рассчитаны поправки к массам частиц и заряду во втором и третьем порядках по константе взаимодействия соответственно. Показано, что полученные поправки выражаются через ковариантные величины, не зависят от импульсов соответствующих частиц и воспроизводят результаты, полученные методом Дайсона - Фейнмана.

МЕЗОН-ДВОНУКЛОННІ ОДЯГНЕНІ ГЕНЕРАТОРИ ГРУПИ ПУАНКАРЕ У КВАНТОВІЙ ТЕОРІЇ ПОЛЯ: ПЕРЕНОРМУВАННЯ МАСИ ТА ЗАРЯДУ

В. Ю. Корда, П. О. Фролов

Методом унітарних одягаючих перетворень реалізовано представлення ”одягнених” частинок, у якому генератори групи Пуанкаре набувають одну й ту ж саму розріджену структуру у фоківському просторі станів частинок. У моделі взаємодіючих за допомогою трилінійного псевдоскалярного зв’язку типу Юкави нуклонного й нейтрального піонного полів розраховано зсуви мас частинок і заряду в другому й третьому порядках за константою взаємодії відповідно. Доведено, що розраховані поправки містять коваріантні величини, не залежать від імпульсів відповідних частинок і відтворюють результати, знайдені за методом Дайсона - Фейнмана.