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APPLICATION OF THE UNITARY CLOTHING TRANSFORMATION METHOD TO THE CHARGE RENORMALIZATION PROBLEM IN QUANTUM FIELD THEORY

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In the toy model of interacting scalar mesons and charged spinless nucleons the renormalization problem in the lowest orders in the coupling constant g is investigated as a byproduct of the so-called "clothing" procedure in the quantum field theory, put forward by Greenberg and Schweber. The expression for the charge shift in the g^3 order is derived. It *naturally* contains the *off-energy-shell* structures and, if projected on the energy shell, can be brought to the explicitly covariant form, providing the momentum independence and another representation for the respective Dyson - Feynman result.

1. Introduction

The consideration of the hadronic systems as the few-body systems faces several fundamental problems which are still relevant and need thorough theoretical exploration on one and the same physical background. Usually, the primary interaction vertex contains particles which can not stay simultaneously on their mass shells. This makes energies of the intermediate states for a given process be absolutely arbitrary and the relativistic effects provide essential contributions to the observable results. The requirement of the relativistic invariance of the theory becomes of the bare necessity for the hadronic systems already at low energies, including the nuclear bound states. Besides, the interaction in the primary form generates the persistent effects after Van Hove [1, 2], which cause the so-called self-interactions and essentially complicates the usual treatment of interaction processes between particles.

It appears that we can divide the same Hamiltonian into a new free part and a new interaction term in a manner which avoids certain of the mentioned difficulties. The desired properties of the Hamiltonian operator can be achieved via the *unitary clothing transformation* method providing the new representation of the Hamiltonian through the "clothed" one-particle operators, which generate the eigenstates of the both free and total Hamiltonians [3].

In the clothed particle representation (CPR), all of the Poincaré group generators are brought to the sparse structure and the commutation relations of the Lie algebra are kept unchanged. Being obtained on one and the same physical footing, the operators of physical interactions between physical particles are Hermitian, energy independent and contain the off-energy-shell structures in a natural way. While implementing the "clothing" procedure, the so-called "bad" terms in the Hamiltonian, starting from the primary interaction operator, which prevent the one particle states to be the eigenstates of the total Hamiltonian, can be eliminated order by order in the coupling constant.

In the field theoretical model of interacting scalar mesons and charged spinless nucleons we derive the expression for the charge shift in the first non-vanishing (third) order in coupling constant as the byproduct of the clothing procedure. It is important to note, that since the off-energy-shell structures are contained in the obtained expression, the latter can be divided into two parts, i.e., the "off-energy-shell" part which goes to zero on the energy shell and the "Feynman-like" part which, being projected on the energy shell, gives an explicitly covariant result coinciding with the correspondent Dyson - Feynman result.

2. The field theoretical model

We start from the representation of *"bare" particles with physical masses* [4] in the field theoretical model of *scalar mesons and charged spinless nucleons* coupled via the three-linear Yukawa-type interaction. The total Hamiltonian can be presented in the following form:

$$H = H_F + H_I = H_F + V + M_{ren} + V_{ren}\,, \tag{1}$$

where H_F is the free part of Hamiltonian, which has the form

$$H_F = \int d^3q E_q \sum_{i=-1,1} F_q^{i\dagger} F_q^i + \int d^3k \omega_k a_k^\dagger a_k\,, \tag{2}$$

and V is the primary interaction operator

$$V = -\frac{g}{(2\pi)^{3/2}} \int \frac{d^3 p d^3 q d^3 k}{\left(8E_p E_q \omega_k\right)^{1/2}} \delta\left(\vec{p} - \vec{q} + \vec{k}\right) \sum_{i,j} F_p^{i\dagger} F_q^j a_k^{\dagger} + H.c.. \tag{3}$$

The mass counterterm M_{ren} is divided into two parts

$$M_{ren} = M_{ren,mes} + M_{ren,nucl}, \tag{4}$$

where the mesonic mass shift $\delta\mu^2 = \mu_0^2 - \mu^2$ enters the correspondent mesonic mass counterterm

$$M_{ren,\,mes} = \frac{\delta\mu^2}{4} \int \frac{d^3 k}{\omega_k} \left(a_k^{\dagger} a_k + a_k^{\dagger} a_{-k}^{\dagger} \right) + H.c., \tag{5}$$

while the nucleonic counterterm depends on the nucleonic mass shift $\delta m^2 = m_0^2 - m^2$:

$$M_{ren,nucl} = \frac{\delta m^2}{4} \int \frac{d^3 q}{E_q} \sum_{i,j=-1,1} F_q^{i\dagger} F_q^j . \tag{6}$$

The vertex counterterm V_{ren}, containing the charge shift $\delta g = g_0 - g$, is given by the formula

$$V_{ren} = -\frac{\delta g}{(2\pi)^{3/2}} \int d^3 p d^3 q d^3 k \frac{\delta\left(\vec{p} - \vec{q} + \vec{k}\right)}{\left(8E_p E_q \omega_k\right)^{1/2}} \sum_{i,j=-1,1} F_p^{i\dagger} F_q^j a_k^{\dagger} + H.c. \tag{7}$$

In Eqs. (2) - (7) the following notations are adopted: $\omega_k = \sqrt{\mu^2 + \vec{k}^2}$ is the energy of physical meson, $E_q = \sqrt{m^2 + \vec{q}^2}$ is the energy of physical nucleon; μ, m and g are the physical masses and the coupling constant respectively, while m_0, μ_0 and g_0 are their respective bare partners. The operators $F_q^{i\dagger}$ (F_q^i) unify the nucleonic $b_q^{\dagger}$ (b_q) and antinucleonic $d_q^{\dagger}$ (d_q) creation (destruction) operators in the following way:

$$F_q^{i\dagger} = \begin{cases} b_q^{\dagger} & i = 1, \\ d_{-q} & i = -1, \end{cases} \quad F_q^i = \begin{cases} b_q & i = 1, \\ d_{-q}^{+} & i = -1, \end{cases} \tag{8}$$

with the commutation relations

$$\left[F_p^i, F_q^{j\dagger}\right] = i\delta_{ij}\delta\left(\vec{p} - \vec{q}\right), \tag{9}$$

which follow from the conventional commutation relations for the bosonic operators $b_q^{\dagger}$ (b_q) and $d_q^{\dagger}$ (d_q): $\left[b_p, b_q^{\dagger}\right] = \delta\left(\vec{p} - \vec{q}\right)$ and $\left[d_p, d_q^{\dagger}\right] = \delta\left(\vec{p} - \vec{q}\right)$. The same relation is valid for the mesonic creation (destruction) operators $a_k^{\dagger}$ (a_k)

$$\left[a_k, a_{k'}^{\dagger}\right] = \delta\left(\vec{k} - \vec{k}'\right). \tag{10}$$

3. Unitary clothing transformation formalism

By definition, the state of "bare" vacuum Ω_0 and the one-bare-particle states $a_k^{\dagger}\Omega_0$, $b_q^{\dagger}\Omega_0$ and $d_q^{\dagger}\Omega_0$ are the eigenstates of the free Hamiltonian H_F. However, they are not the eigenstates of the total Hamiltonian H because the latter contains the terms, called "bad", of the form, e.g., $a^{\dagger}a^{\dagger}$, $b^{\dagger}d^{\dagger}$, $b^{\dagger}d^{\dagger}a$, etc. and their adjoint counterparts, which enter the operators V, M_{ren}, and V_{ren}.

To ameliorate the problem, we shall try to eliminate these terms from the Hamiltonian by making the transition to the CPR which can be provided by means of the unitary clothing transformation applied to the whole set of particle operators

$$\alpha_c = W^\dagger \alpha W \ , \ WW^\dagger = W^\dagger W = 1 , \tag{11}$$

where α_c and α denote the sets of the clothed (physical) particle operators and the bare (primary) particle operators with physical masses respectively.

In terms of new (clothed) particle operators the total Hamiltonian reaches the form

$$H = H(\alpha) = H\left(W^\dagger(\alpha_c)\alpha_c W(\alpha_c)\right) = W(\alpha_c) H(\alpha_c) W^\dagger(\alpha_c) =$$

$$= H_F(\alpha_c) + H_I(\alpha_c) + [R, H_F] + [R, H_I] + \frac{1}{2}[R,[R,H_F]] + \frac{1}{2}[R,[R,H_I]] + \dots , \tag{12}$$

or

$$H = H_F + V + M_{ren} + V_{ren} +$$

$$+ [R, H_F] + [R,V] + [R, M_{ren}] + [R, V_{ren}] + \frac{1}{2}[R,[R,H_F]] + \frac{1}{2}[R,[R,V]] + \dots \tag{13}$$

where

$$W = W(\alpha) = W(\alpha_c) = \exp\left(R(\alpha_c)\right), \ R(\alpha_c) = -R^\dagger(\alpha_c) . \tag{14}$$

The requirement put over the generator R is the elimination of bad terms from the Hamiltonian (13). In the wide class of quantum field models the interaction operator is the g^1- order bad term found in the Hamiltonian. Therefore, we look for the generator of the unitary transformation in the form which satisfies the equation

$$V + [R, H_F] = 0 . \tag{15}$$

The generator R determined from Eq. (15) has the same operator structure as the interaction operator V does. In the CPR, one gets (for brevity, further we drop the subscript c for all the clothed operators):

$$R = \frac{g}{(2\pi)^{3/2}} \int \frac{d^3p d^3q d^3k}{\left(8E_p E_q \omega_k\right)^{1/2}} \delta\left(\vec{p} - \vec{q} + \vec{k}\right) \sum_{i,j} \left(D^{i,-j}_{p,q,k} F^{i\dagger}_p F^j_q a^\dagger_k\right) - H.c. , \tag{16}$$

where we adopt the denotation: $D^{i,-j}_{p,q,k} = \dfrac{1}{iE_p - jE_q + \omega_k}$.

Taking into account equation (15), the Hamiltonian (13) can be written in the following form:

$$H = H_F + M_{ren} + \frac{1}{2}[R,V] + V_{ren} + [R, M_{ren}] + \frac{1}{3}[R,[R,V]] + \dots . \tag{17}$$

We emphasize that now the total Hamiltonian (17) does not contain bad terms of the g^1-order. To proceed with the clothing procedure to the second order in the coupling constant one should collect and remove all of the respective bad terms from Eq. (17). In particular, it allows us to obtain the interaction operators for the nucleon-nucleon and meson-nucleon scattering processes in the g^2-order. Further, the removal of the g^3 bad terms gives the interaction operators for the meson production and absorption on the pair of nucleons in the g^3-order. Simultaneously, as a byproduct, we obtain the expression for the mass and charge shifts in their first non-vanishing orders in g.

4. Mass renormalization

The operators $M_{ren,nucl}$ and $M_{ren,mes}$ containing the nucleonic and mesonic mass shifts consist of the operators the structure of which are inherent in some operators appearing in the g^2-order commutator $[R, V]$. Really, using Eqs. (3) and (16) we get

$$[R,V]=-\frac{g^2}{(2\pi)^3}\int\frac{d^3pd^3qd^3kd^3p'd^3q'd^3k'}{8\left(E_pE_qE_{p'}E_{q'}\omega_k\omega_{k'}\right)^{1/2}}\delta\left(\vec{p}'-\vec{q}'+\vec{k}'\right)\delta\left(\vec{p}-\vec{q}+\vec{k}\right)\times$$

$$\times\sum_{i,i',j,j'}\left\{j\delta_{ji'}\delta(\vec{q}-\vec{p}')F_p^{i\dagger}F_{q'}^{j'}a_k^\dagger a_{k'}^\dagger\left(D_{p,q,k}^{i,-j}-D_{p',q',k'}^{i',-j'}\right)+j\delta_{jj'}\delta(\vec{q}-\vec{q}')\left(F_{p'}^{i'\dagger}F_p^i a_{k'}^\dagger a_k\, D_{p,q,k}^{i,-j}-F_{p'}^{i'\dagger}F_p^i a_{-k}^\dagger a_{-k'}\, D_{p',q',k'}^{-i',j'}\right)-\right.$$

$$\left.-\delta\left(\vec{k}-\vec{k}'\right)F_q^{j\dagger}F_p^iF_{p'}^{i'\dagger}F_{q'}^{j'}\, D_{p,q,k}^{i,-j}\right\}+H.c.. \tag{18}$$

After normal ordering in (18) we are able to separate the two-meson and two-nucleon operator parts from $[R,V]$

$$[R,V]=:[R,V]:+[R,V]_{2mes}+[R,V]_{2nucl}, \tag{19}$$

with

$$[R,V]_{2mes}=-\frac{g^2}{(2\pi)^3}\int\frac{d^3qd^3k}{8E_{q-k}E_q\omega_k}\left(a_k^\dagger a_{-k}^\dagger+a_k^\dagger a_k\right)\left(D_{q-k,q,k}^{-1,-1}-D_{q-k,q,k}^{1,1}\right)+H.c., \tag{20}$$

$$[R,V]_{2nucl}=\frac{g^2}{(2\pi)^3}\int\frac{d^3qd^3k}{8E_{q-k}E_q\omega_k}\sum_{i,i'}F_{q-k}^{i\dagger}F_{q-k}^{i'}\left(D_{q-k,q,k}^{-1,1}+D_{q-k,q,k}^{1,1}\right)+H.c.. \tag{21}$$

Comparing expressions (5) and (20) in the Hamiltonian, we derive the mesonic mass shift in the second order in the coupling constant

$$\mu_0^2-\mu^2=-\frac{g^2}{(2\pi)^3}\int\frac{d^3q}{4E_qE_{q-k}}\left(D_{q-k,q,k}^{1,1}-D_{q-k,q,k}^{-1,-1}\right). \tag{22}$$

This expression can be interpreted through the mesonic self-energy diagrams (Fig. 1) where the directions of arrows differ particles from antiparticles. These diagrams correspond to the non-covariant propagators in Eq. (22). Namely, the first propagator in Eq. (22) corresponds to the diagram b, while the second propagator corresponds to the diagram a.

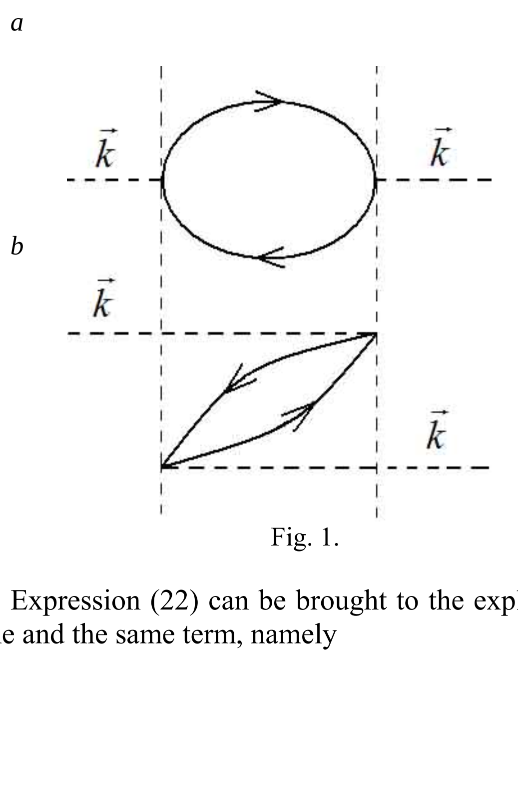

Fig. 1.

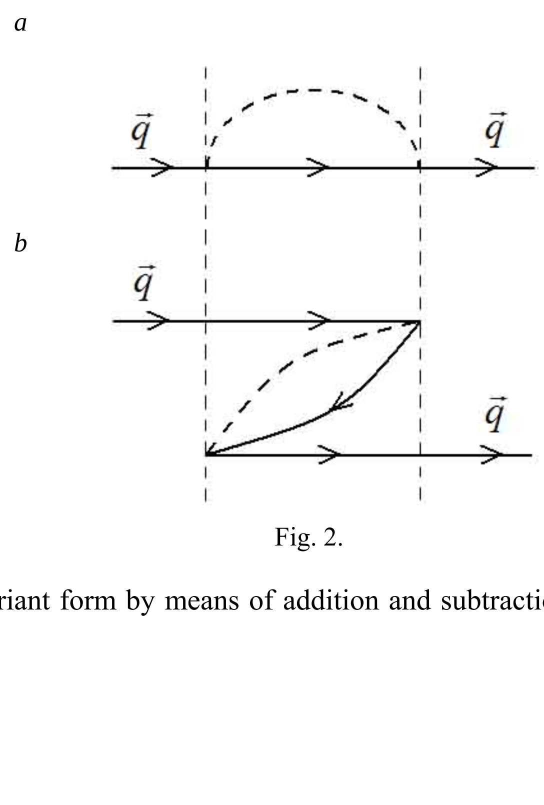

Fig. 2.

Expression (22) can be brought to the explicitly covariant form by means of addition and subtraction of one and the same term, namely

$$\mu_0^2-\mu^2=-\frac{g^2}{(2\pi)^3}\int\frac{d^3q}{4E_qE_{q-k}}\times$$

$$\times\left[\left(\frac{1}{E_{q-k}+E_q+\omega_k}+\frac{1}{E_{q-k}-E_q-\omega_k}\right)-\left(\frac{1}{E_{q-k}-E_q-\omega_k}-\frac{1}{E_{q-k}+E_q-\omega_k}\right)\right]. \tag{23}$$

Finally, unifying two first and two second terms in Eq. (23), the mesonic mass shift in the g^2-order acquires the form (cf., [5]):

$$\delta\mu^2=\frac{g^2}{(2\pi)^3}\int\frac{d^3q}{E_q}\left(\frac{\mu^2}{\mu^4-4(qk)^2}\right), \tag{24}$$

where $q=(E_q,\vec{q})$ and $k=(\omega_k,\vec{k})$.

This expression provides the explicit momentum independence of the mesonic mass shift in the g^2-order and gives another representation of the respective Feynman 4-dimentional covariant integral.

The nucleonic mass shift can be derived in the similar way

$$m_0^2-m^2=-\frac{g^2}{(2\pi)^3}\int\frac{d^3k}{2E_{q-k}\omega_k}\left(D^{1,-1}_{q-k,q,k}+D^{1,1}_{q-k,q,k}\right). \tag{25}$$

Here the first non-covariant propagator corresponds to the diagram *b* on Fig. 2 and the second propagator corresponds to the diagram *a* there. Adding and subtracting one and same term

$$m_0^2-m^2=\frac{g^2}{(2\pi)^3}\int\frac{d^3k}{2E_{q-k}\omega_k}\times$$

$$\times\left[\left(\frac{1}{E_{q-k}+E_q+\omega_k}+\frac{1}{-E_{q-k}-E_q+\omega_k}\right)+\left(-\frac{1}{-E_{q-k}-E_q+\omega_k}+\frac{1}{E_{q-k}-E_q+\omega_k}\right)\right], \tag{26}$$

we bring the expression (25) to the evidently covariant form (cf., [6])

$$\delta m^2=\frac{g^2}{(2\pi)^3}\left(\int\frac{d^3k}{\omega_k}\,\frac{1}{-\mu^2+2qk}+\int\frac{d^3k}{E_k}\,\frac{1}{\mu^2-2m^2+2qk}\right). \tag{27}$$

Expression (27) is just another representation of the respective Feynman covariant result.

5. Charge renormalization

The operator V_{ren} containing the charge shift and the operator $[R,M_{ren}]$ containing the mass shifts (starting from the g^2-order) both consist of the operators the structure of which replicates the structure of some operators appearing in the g^3-order commutator $[R,[R,V]]$. Really, using Eqs. (5), (6) and (16), we find

$$\left[R,M_{ren,\,mes}\right]=\frac{g\left(\mu_0^2-\mu^2\right)}{2(2\pi)^{1/2}}\int\frac{d^3qd^3pd^3k\,\delta\left(\vec{p}-\vec{q}+\vec{k}\right)}{\left(8E_qE_p\omega_k\right)^{1/2}}\times$$

$$\times\sum_{i,j}\frac{1}{\omega_k}\left\{D^{j,-i}_{q,p,k}+D^{i,-j}_{p,q,k}\right\}F_p^{i\dagger}F_q^{j}a_k^{\dagger}+H.c., \tag{28}$$

$$\left[R, M_{ren,\,nucl}\right] = \frac{g\left(m_0^2 - m^2\right)}{4\left(2\pi\right)^{3/2}} \int \frac{d^3q d^3p d^3k\; \delta\left(\vec{p} - \vec{q} + \vec{k}\right)}{\left(8E_q E_p \omega_k\right)^{1/2}} \times$$

$$\times \sum_{i,j,j'} j' \left\{ -\frac{1}{E_q} D_{p,q,k}^{i,-j'} + \frac{1}{E_p} D_{p,q,k}^{j',-j} \right\} F_p^{i\dagger} F_q^j a_k^\dagger + H.c.. \tag{29}$$

At the same time, after some algebra and normal ordering, we are able to separate the two-nucleon-one-meson operator parts from $\left[R,\left[R,V\right]\right]$

$$\left[R,\left[R,V\right]\right]_{2nucl,1mes} = \left[R,\left[R,V\right]\right]_{WFren,mes} + \left[R,\left[R,V\right]\right]_{WFren,nucl} + \left[R,\left[R,V\right]\right]_{Vren}, \tag{30}$$

where the meson and nucleon wave function renormalization terms have the form

$$\left[R,\left[R,V\right]\right]_{WFren,\,mes} = -\frac{3g^3}{\left(2\pi\right)^{9/2}} \int \frac{d^3q d^3q' d^3p d^3k\; \delta\left(\vec{p} - \vec{q} + \vec{k}\right)}{\left(8E_q E_p \omega_k\right)^{1/2} \left(8E_{p-q'} E_{q-q'} \omega_k\right)} \times$$

$$\times \sum_{i,j} \left\{ -D_{q,p,k}^{j,-i} D_{p-q',q-q',k}^{-1,-1} + D_{q,p,k}^{j,-i} D_{p-q',q-q',k}^{1,1} - \right.$$

$$\left. -D_{p-q',q-q',k}^{-1,-1} D_{p,q,k}^{i,-j} + D_{p-q',q-q',k}^{1,1} D_{p,q,k}^{i,-j} \right\} F_p^{i\dagger} F_q^j a_k^\dagger + H.c., \tag{31}$$

$$\left[R,\left[R,V\right]\right]_{WFren,\,nucl} = -\frac{3g^3}{\left(2\pi\right)^{9/2}} \int \frac{d^3q d^3k' d^3p d^3k\; \delta\left(\vec{p} - \vec{q} + \vec{k}\right)}{\left(8E_q E_p \omega_k\right)^{1/2}} \times$$

$$\times \sum_{i,j,j'} j' F_p^{i\dagger} F_q^j a_k^\dagger \left[\frac{1}{8E_{q-k'} E_q \omega_{k'}}\; D_{p,q,k}^{i,-j'} \left(-D_{q-k',q,k'}^{1,-1} - D_{q,q-k',k'}^{1,1} \right) + \right.$$

$$\left. \frac{1}{8E_{p-k'} E_p \omega_{k'}} D_{p,q,k}^{j',-j} \left(D_{p,p-k',k'}^{1,1} + D_{p-k',p,k'}^{1,-1} \right) \right] + H.c., \tag{32}$$

and the vertex renormalization contribution is written as

$$\left[R,\left[R,V\right]\right]_{Vren} = -\frac{g^3}{\left(2\pi\right)^{9/2}} \int \frac{d^3q d^3k' d^3p d^3k\; \delta\left(\vec{p} - \vec{q} + \vec{k}\right)}{\left(512 E_q E_p \omega_k\right)^{1/2} E_{p-k'} E_{q-k'} \omega_{k'}} \times$$

$$\times \sum_{i,j} F_p^{i\dagger} F_q^j a_k^\dagger \left\{ -D_{q,q-k',k'}^{i,-1} D_{p-k',q-k',k}^{-1,-1} + 2D_{q,q-k',k'}^{i,1} D_{p-k',q-k',k}^{1,1} - \right.$$

$$-D_{p-k',q-k',k}^{-1,1} D_{q,q-k',k'}^{i,1} + D_{q-k',q,k'}^{-1,-i} D_{p-k',q-k',k}^{1,1} -$$

$$-2D_{q-k',q,k'}^{1,-i} D_{p-k',q-k',k}^{-1,-1} + D_{p-k',q-k',k}^{1,-1} D_{q-k',q,k'}^{1,-i} -$$

$$-D_{p-k',p,k'}^{-1,-j} D_{p-k',q-k',k}^{-1,-1} + 2D_{p-k',p,k'}^{1,-j} D_{p-k',q-k',k}^{1,1} -$$

$$-D^{1,-1}_{p-k',q-k',k}D^{1,-j}_{p-k',p,k'}+D^{j,-1}_{p,p-k',k'}D^{1,1}_{p-k',q-k',k}-$$

$$-2D^{j,1}_{p,p-k',k'}D^{-1,-1}_{p-k',q-k',k}+D^{-1,1}_{p-k',q-k',k}D^{j,1}_{p,p-k',k'}+$$

$$+2D^{1,-i}_{q-k',q,k'}D^{1,-j}_{p-k',p,k'}+2D^{j,1}_{p,p-k',k'}D^{i,1}_{q,q-k',k'}-$$

$$-D^{j,-1}_{p,p-k',k'}D^{i,1}_{q,q-k',k'}-D^{j,1}_{p,p-k',k'}D^{i,-1}_{q,q-k',k'}-$$

$$-D^{-1,-i}_{q-k',q,k'}D^{1,-j}_{p-k',p,k'}-D^{1,-i}_{q-k',q,k'}D^{-1,-j}_{p-k',p,k'}\Big\}+H.c.. \quad (33)$$

Bearing in mind that the operators in the formula (30) are of the g^3-order while the operators in the formulae (28) and (29) start from the g^3-order, we find the following correlations:

$$\left[R,M_{ren,mes}\right]+\frac{1}{3}\left[R,\left[R,V\right]\right]_{WFren,mes}=0\,, \quad (34)$$

$$\left[R,M_{ren,\,nucl}\right]+\frac{1}{3}\left[R,\left[R,V\right]\right]_{WFren,nucl}=0\,, \quad (35)$$

valid in the g^3-order.

Further, it is convenient to rewrite the expression (33) in the form

$$\left[R,\left[R,V\right]\right]_{Vren}=\int d^3q d^3k' d^3p d^3k \sum_{i,j} G^{i,j}_{p,q,k,k'}F^{i\dagger}_p F^j_q a^\dagger_k + H.c.. \quad (36)$$

It can be immediately verified that under the integration in Eq. (36)

$$G^{1,1}_{p,q,k,k'}=G^{-1,-1}_{p,q,k,k'}\,. \quad (37)$$

Taking into account the equality (36) and comparing the operators V_{ren} and $\left[R,\left[R,V\right]\right]_{Vren}$ in the Hamiltonian, we find the following expression for the charge shift in the g^3-order:

$$\delta g=\frac{g^3}{24\left(2\pi\right)^{9/2}}\int\frac{d^3k'}{E_{p-k'}E_{q-k'}\omega_{k'}}\times$$

$$\times\Big\{D^{1,-1}_{q-k',q,k'}D^{1,-1}_{p-k',q-k',k}+2D^{1,-1}_{q-k',q,k'}D^{1,-1}_{p-k',p,k'}-D^{1,-1}_{p-k',q-k',k}D^{1,-1}_{p-k',p,k'}-$$

$$-2D^{1,-1}_{q-k',q,k'}D^{-1,-1}_{p-k',q-k',k}-D^{1,-1}_{q-k',q,k'}D^{-1,-1}_{p-k',p,k'}-D^{-1,-1}_{p-k',p,k'}D^{-1,-1}_{p-k',q-k',k}-$$

$$-D^{1,1}_{q,q-k',k'}D^{-1,1}_{p-k',q-k',k}+2D^{1,1}_{q,q-k',k'}D^{1,1}_{p,p-k',k'}+D^{1,1}_{p,p-k',k'}D^{-1,1}_{p-k',q-k',k}+$$

$$+D^{-1,-1}_{q-k',q,k'}D^{1,1}_{p-k',q-k',k}-D^{-1,-1}_{q-k',q,k'}D^{1,-1}_{p-k',p,k'}+2D^{1,-1}_{p-k',p,k'}D^{1,1}_{p-k',q-k',k}-$$

$$-D^{1,-1}_{q,q-k',k'}D^{-1,-1}_{p-k',q-k',k}-D^{1,-1}_{q,q-k',k'}D^{1,1}_{p,p-k',k'}-2D^{1,1}_{p,p-k',k'}D^{-1,-1}_{p-k',q-k',k}+$$

$$+2D^{1,1}_{q,q-k',k'}D^{1,1}_{p-k',q-k',k}-D^{1,1}_{q,q-k',k'}D^{1,-1}_{p,p-k',k'}+D^{1,-1}_{p,p-k',k'}D^{1,1}_{p-k',q-k',k}\Big\}. \quad (38)$$

In Eq. (38) each of the eighteen products of the non-covariant propagators corresponds to the one of the six graphs shown in Fig. 3 (as before, arrows differ particles from antiparticles). For instance, first three terms in (38) are displayed by the graph *a*, second three terms are illustrated by the graph *b*, etc.

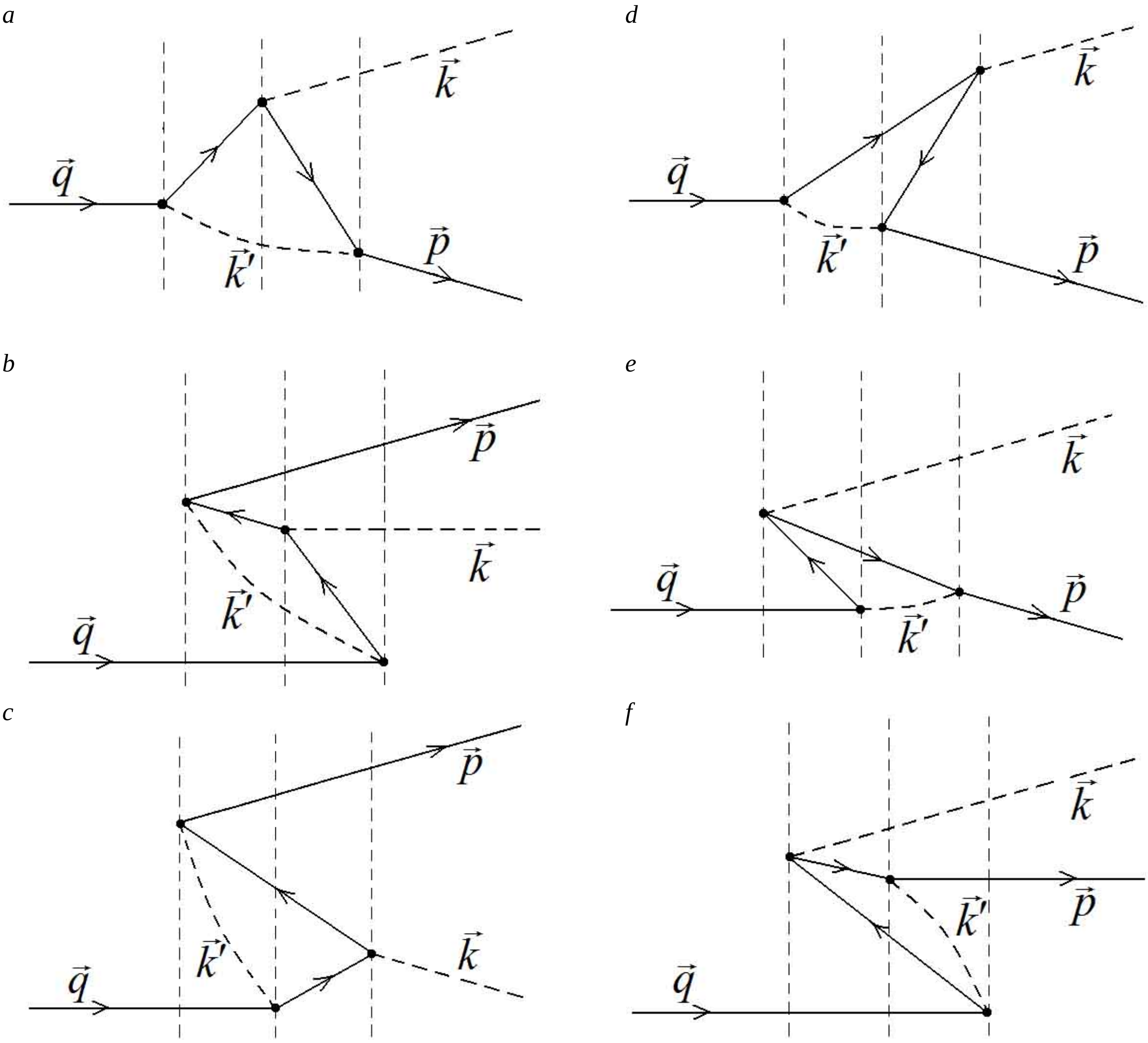

Fig. 3.

It is important that the formula (38) permits the following division:

$$\delta g = \delta g_{Feynman-like} + \delta g_{off-energy-shell}, \tag{39}$$

where the "Feynman-like" part is on the energy shell while the "off-energy-shell" part disappears on the energy shell. The Feynman-like correction is determined by the formula, where we assume the transparent notation $D^{i,j,j'}_{p,q,k,k'} = \dfrac{1}{iE_p + jE_q + j'\omega_k + \omega_k}$:

$$\delta g_{Feynman-like} = \frac{g^3}{8(2\pi)^{9/2}} \int \frac{d^3k' \; \delta\left(\vec{p} - \vec{q} + \vec{k}\right)}{E_{p-k'} E_{q-k'} \omega_{k'}} \times$$

$$\times \Big\{ \left(D^{-1,1}_{q-k,q,k} + D^{1,-1}_{q-k,q,k} \right) \left(D^{-1,1}_{q,p'',q-p''} - D^{-1,-1}_{q,p'',q-p''} \right) +$$

$$+ \left(D^{1,1}_{q,p'',q-p''} - D^{-1,1}_{q,p'',q-p''} \right) \left(D^{-1,1,1}_{q-k,p'',q-p'',k} + D^{1,-1,-1}_{q-k,p'',q-p'',k} \right) +$$

$$+ \left(D^{1,1}_{q-k,q,k} - D^{1,-1}_{q-k,q,k} \right) \left(D^{1,1,-1}_{q-k,p'',q-p'',k} - D^{1,-1,-1}_{q-k,p'',q-p'',k} \right) \Big\}, \tag{40}$$

which can be brought to the explicitly covariant form

$$\delta g_{Feynman-like} = \frac{1}{2}\frac{g^3}{(2\pi)^3}\left[\int \frac{d^3p'}{E_{p'}}\left(\frac{1}{\left(\mu^2 - 2p'k\right)\left(\mu^2 - 2m^2 - 2p'p\right)}\right) - \right.$$

$$\left. -\int \frac{d^3k'}{\omega_{k'}}\left(\frac{1}{\left(\mu^2 + 2k'p\right)\left(\mu^2 + 2k'q\right)}\right) + \int \frac{d^3p'}{E_{p'}}\left(\frac{1}{\left(\mu^2 + 2p'k\right)\left(\mu^2 - 2m^2 + 2p'q\right)}\right)\right], \qquad (41)$$

providing the momentum independence and another representation of the corresponding Dyson-Feynman result. Here $p = \left(E_p, \vec{p}\right)$, $p' = \left(E_{p'}, \vec{p}'\right)$, $k' = \left(\omega_{k'}, \vec{k}'\right)$.

Evidently, the actual value for the charge shift must be calculated with help of Eq. (41). This means that the respective g^3 part of the operator $\left[R,\left[R,V\right]\right]_{Vren}$ which involves the off-energy shell contribution in the formula (39) must enter the list of the g^3-order bad terms and be removed by the forthcoming clothing unitary transformation.

6. Conclusion

It is important to emphasize that, as we have shown the problem of mass and charge renormalization in the clothed particle representation in the quantum field theory is solved automatically as a byproduct of the application of the unitary clothing transformation procedure. Being utilized order by order in the coupling constant, the latter cleans out the total Hamiltonian from the operators which prevent the one-particle-states to be its eigenstates, reaching the very goal of Greenberg and Schweber. If the procedure is believed to be successfully accomplished in all orders then the total Hamiltonian shall contain only the operators of physical interactions between physical (observed) particles, that can be used for solving the respective relativistic dynamical equations.

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ПРИМЕНЕНИЕ МЕТОДА УНИТАРНЫХ ОДЕВАЮЩИХ ПРЕОБРАЗОВАНИЙ К ПРОБЛЕМЕ ПЕРЕНОРМИРОВКИ ЗАРЯДА В КВАНТОВОЙ ТЕОРИИ ПОЛЯ

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На примере простой модели взаимодействующих скалярных мезонов и заряженных бесспиновых нуклонов проблема перенормировок в первых порядках по константе взаимодействия исследована с помощью предложенной Гринбергом и Швебером так называемой процедуры “одевания” в квантовой теории поля. Рассчитанная поправка третьего порядка к величине заряда содержит в естественной форме структуры вне энергетической оболочки и приводится к ковариантному виду на энергетической оболочке, обеспечивая независимость сдвига заряда от импульсов частиц и предлагая другое представление для соответствующего результата, полученного методом Дайсона - Фейнмана.

ЗАСТОСУВАННЯ МЕТОДУ УНІТАРНИХ ОДЯГАЮЧИХ ПЕРЕТВОРЕНЬ ДО ПРОБЛЕМИ ПЕРЕНОРМУВАННЯ ЗАРЯДУ У КВАНТОВІЙ ТЕОРІЇ ПОЛЯ

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На прикладі простої моделі взаємодіючих скалярних мезонів і заряджених безспінових нуклонів проблема перенормування в перших порядках за константою взаємодії досліджена за допомогою запропонованої Грінбергом і Швебером так званої процедури “одягання” у квантовій теорії поля. Розрахована поправка третього порядку до величини заряду містить у природній формі структури поза енергетичною оболонкою й приводиться до коваріантного вигляду на енергетичній оболонці, забезпечуючи незалежність зсуву заряду від імпульсів частинок і пропонуючи інше представлення для відповідного результату, знайденого методом Дайсона - Фейнмана.