

## NEW COLOR STATISTICS FOR THE HEAVY NUCLEI FISSION FRAGMENT SYSTEMATIZATION

**V. T. Maslyuk, O. O. Parlag, O. I. Lendyel, T. J. Marinetc**

*Institute of Electron Physics, National Academy of Sciences Ukraine, Uzhgorod, Ukraine*

A new statistical method for systematizing the heavy nuclei fission fragments and investigating their mass and charge spectra, which follows from studying stability conditions of the post-fission ensemble of nuclear fragments, is suggested. In this method, protons (neutrons) of different fragments are considered statistically non-equivalent. The possibility to apply new statistics to explain known experimental data on heavy nuclei fission is shown.

### 1. Introduction

It is known well that the mass (charge) spectra of atomic nuclei fission fragments (MCSFF) depend both on their formula (numbers, proton and neutron ratios) and excitation energies [1]. Presence of symmetric or asymmetric MCSFF shape and fragments symmetrization with increasing excitation energy and atomic mass is a known nuclear physics puzzle, which finds no explanation in terms of quantum nuclear models. This situation is similar to that in the solid-state physics, where the progress in *ab initio* calculations of fragment electron characteristics is not extended, e.g., to the explanation of the mechanism of aggregative state change.

Present paper shows that MCSFF peculiarities could be explained within the framework of new statistical approach, in which the subject of the studies is the post-fission state of the nucleus derived from the condition of thermodynamical equilibrium of the final nuclear fragment ensemble. The structure of this paper is as follows: the first section deals with the principles and assumptions laid in the basis of this approach, the second section illustrate the abilities to interpret known experiments (see [2 - 4] and predictions of fission characteristics of heavy nuclei). Conclusion deals with the discussion of the possibilities of the suggested statistical approach application.

### 2. Theory

In the suggested approach, the nuclei is considered the condensed state of the system of nucleons with constant volume, while the binding energy is non-additive with respect to the sort and number of nucleon particles. The object of the studies is the state of deformed nucleus not before but after passing the saddle point, when the nucleon composition of fragment nuclei is formed, though the thermodynamical parameters (temperature T, pressure P) are the same for each fragment. In this case the problem of finding MCSFF is reduced to the analysis of a constant pressure canonic ensemble formed when taking into account any possible scheme of primary nucleus decay and fission neutrons emission.

Let us restrict ourselves to the scheme of two-fragment heavy nuclei fission. Then the *i*-th cluster of nuclear fragments includes  $Z_{j,i}$  protons,  $A_{j,i} - Z_{j,i}$  neutrons in the *j*-th fragment,  $j = 1, 2$ , and  $n_i$  fission neutrons, for which the conservation conditions are valid:  $A_{1,i} + A_{2,i} + n_i = A_0$ ,  $Z_{1,i} + Z_{2,i} = Z_0$ . It is known well that fission neutron emission, besides affecting the energetics of heavy nuclei fission, results in the change of the volume V of the primary nucleus [1], which, given constant pressure of neutron gas in the nucleus, is accompanied by execution of a work. Thus, the condition of the fission fragment equilibrium shall be derived from the Gibbs thermodynamical potential minimum

$$G = U - TS + PV, \quad (1)$$

here S is the configuration entropy, while other parameters are explained above.

When performing analysis (1), one may assume [5] that, with accuracy to a constant, the internal energy U is an additive quantity with respect to the binding energy of nuclear fragments,

$$U_{\{i\}} = \sum_{j=1,2} \cdot \sum_{\langle N_p \rangle} \cdot \sum_{\langle N_n \rangle} U_j(A_{j,i}, Z_{j,i}), \quad (2)$$

where  $U_j$  is the binding energy of the *j*-th fission fragment,  $j = 1, 2$ , symbol  $\langle \dots \rangle_i$  means that summation in (3) is taken over the *i*-th two-fragment clusters, the sets of protons or neutrons in which satisfy the condition  $\sum_{j=1,2} (N_p^j + N_n^j + n_j) = A_0$ .

Configuration entropy  $S_{\{i\}}$  introduced in (1) must be calculated with the allowance made for the statistical non-equivalence of the protons (neutrons) with different specific binding energy in each of nuclear fragment and in the free state for the fission neutrons. Formally, this should be done by introducing the conditional colors, which differentiate twin nucleons (protons, neutrons) of different nuclear fragments. In this case  $S_{\{i\}} = \ln (w_i)$ , where

$$w_i = A_0!K(n_j)/\left(\prod_{j=1,2} (N_p^{(j)}!N_n^{(j)}n_j!)\right), \quad (3)$$

$K(n_j) = 1/n_j!$ ,  $\prod_{j=1,2} x_j! = x_1!x_2!$   $i$ -th number of two-fragment cluster produced due to the initial nucleus fission. The analysis of expression (3) shows that the entropy term will be maximal at  $N_p^j = N_n^j$  and is responsible for symmetrization of the fission fragment yield with increasing nuclear temperature. The statistics, which follows from accounting the statistical non-equivalence of protons (neutrons) of the fission fragment ensemble, could be called the "color" statistics [5].

To obtain the numerical values of MCSFF for different T values and nucleus isotope composition, one has to calculate the isobaric distribution function, which is a probability of finding the two-fragment cluster in the  $i$ -th quantum ensemble with the energy  $\varepsilon_i(V)$ , which has a form

$$f_i(V) = \exp\{-\varepsilon_i(V) + PV\}/kT\} / Z_p. \quad (4)$$

Here  $Z_p$  is the isobaric statistical sum determined from the normalization condition  $\sum_{i=1,N} f_i(V) = 1$ .

Formulae (3) and (4) allow one to calculate MCSFF for heavy nuclei in different approximations for independent yields and cumulative fission fragment yields. The numerical values for the respective yields Y shall be determined by normalizing (onto 200 %) the distribution functions  $f_i(V)$  (7) of the two-fragment clusters found from the condition of thermodynamical potential minimum for the ensemble (1).

The nuclear temperature T in (1) is defined by the excitation energy of the primary nucleus and within the terms of suggested approach is a characteristic of the thermostat of nuclear fragment ensemble, i.e. it is the theory parameter. In practice, the value of T is determined from the fission neutron [6] or proton [7] emission spectra.

### 3. MCSFF experiments and possibility of their explanation

Below we shall discuss the experiments on studying the heavy nuclei MCSFF with increasing primary nucleus excitation energy (see review [2]), i.e. the so-called Darmstadt experiment [3] on the charge spectra of the U, Pa, Th, Ac, Ra isotopes, as well as the experiment [4] on the mass spectra of the Xe, Kr isotopes at the Np, Th, U, Pu nuclei fission.

The idea of calculation lies in the formation of an ensemble of all possible two-fragment clusters, each of them including a certain fission neutron number. Suggested theory relates neutron "evaporation" with thermodynamical parameters equalizing within the ensemble. MCSFF for atomic nuclei are determined from the condition of thermodynamical potential G minimum (1). To do this, one has to process an ensemble including more than 10,000 - 20,000 two-nucleon clusters (for the database of experimental binding energies [8]) with high yield probability over the indicated channels. Besides the nuclear temperature T of the primary nucleus, the parameters of the theory include also the value of isobaric term in (1) chosen in a form of  $PV = P(V_0 - v_0n)$ , where  $V_0$  is the initial volume of the primary nucleus,  $P$  is the nucleon "gas" pressure in the primary nucleus,  $v_0$  is the average volume related to a single fission neutron with total number of  $n$ . The isobaric constant  $Pv_0$  value was 2 - 4 MeV and was chosen from the condition that the average fission neutron number  $\bar{n}$  at  $T \sim 1$  MeV does not exceed  $\bar{n} = 4$  neutron/fission.

**MCSFF summarization with increasing temperature T.** This problem was studied by us using the  $^{237}\text{Np}$  nuclei fission fragments as the example. Fig. 1, *a* shows the energy relief of the potential G in the coordinates of the mass (charge) nuclear fragment number for this isotope, while Fig. 1, *b* illustrates their mass spectrum at T increasing from 0.5 to 2.5 MeV.

As seen, in terms of suggested approach, MCSFF symmetrization is explained by the rise of the entropy factors for the above fission fragment ensemble at high T. This regularity is of universal character, while the conditions of transition from asymmetric MCSFF shape to symmetric one are influenced by the isotope sort and neutron activity during fission.

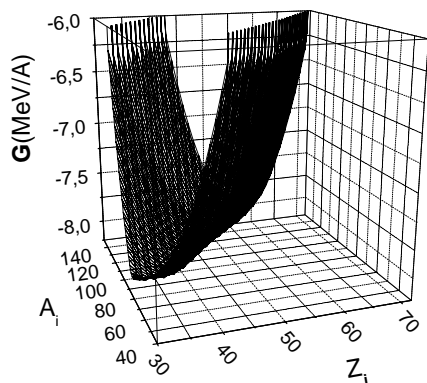


Fig. 1, *a*. Gibbs potential dependence for the  $^{237}\text{Np}$  fission fragment ensemble at  $T = 1$  MeV.

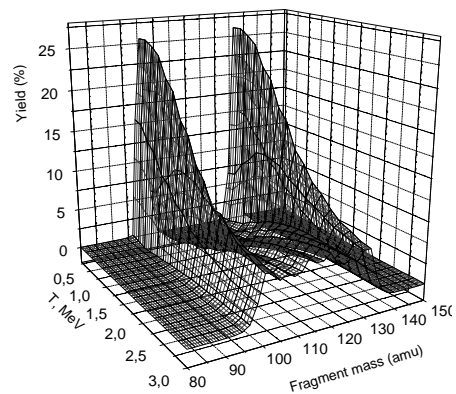


Fig. 1, *b*. Temperature dependence of the  $^{237}\text{Np}$  fission fragment yields.

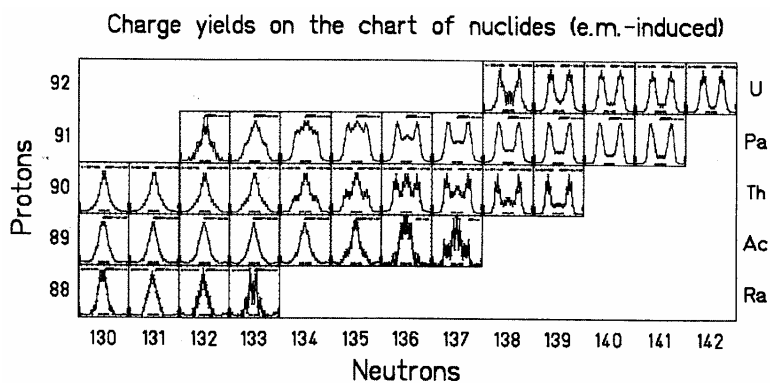


Fig. 2. Experimental charge spectra of the U, Pa, Th, Ac, Ra isotope fission fragments taken from [3].

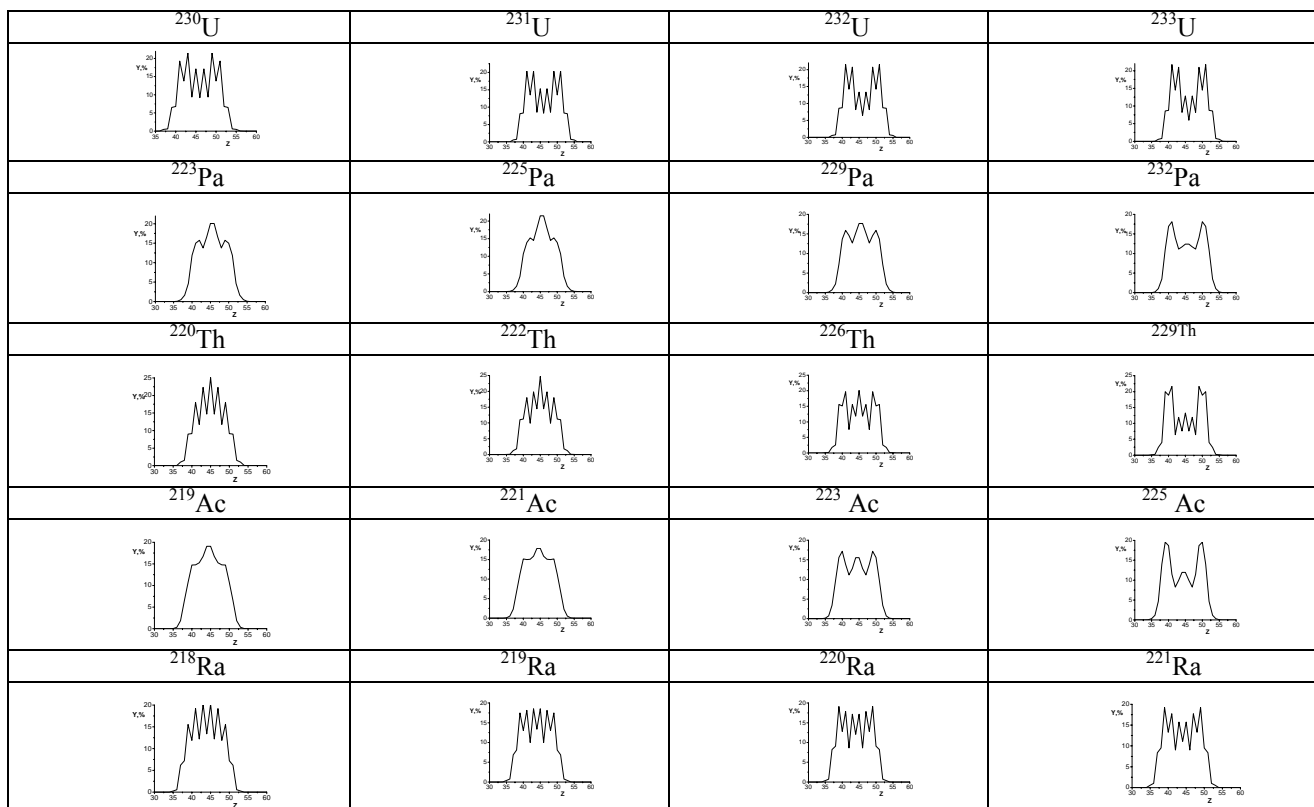


Fig. 3. Charge spectra of the U, Pa, Th, Ac, Ra isotope fission fragments obtained within the framework of suggested statistical method.

**Darmstadt experiment: charge spectra of the U, Pa, Th, Ac, Ra isotope fission fragments.** The charge spectra of the above isotopes were studied in the experiment with radioactive beams formed by the 1A GeV  $^{238}\text{U}$  beam fragmentation at the heavy-ion synchrotron SIS at DESI Darmstadt (see [3] for experimental details). The data obtained (Fig. 2) indicate different shape of charge spectra for the isotopes of the same chemical compound, revelation of a symmetric mode for the proton-rich fissile nuclei and asymmetric modes, as a rule, with increasing mass number. The same tendency comes from the theoretical data (Fig. 3) obtained at  $T = 1.2$  MeV and from the above model parameters. It is seen that theoretical curves reflect the principal regularities of experiment.

**Kr, Xe isotope spectra for the  $^{237}\text{Np}$  photofission.** Fig. 4 presents both experimental (1) and theoretical (2) Kr, Xe isotope fragment yields for the  $^{237}\text{Np}$  photofission. The experimental details are presented in [4], the theoretical dependences were obtained assuming  $T = 0.7$  MeV, the  $P\nu_0$  value was, similarly to the previous cases 2 to 4 MeV. As seen, the curves shown in fig. 4 illustrate a good agreement in the peak positions for the Kr and Xe isotope dependences.

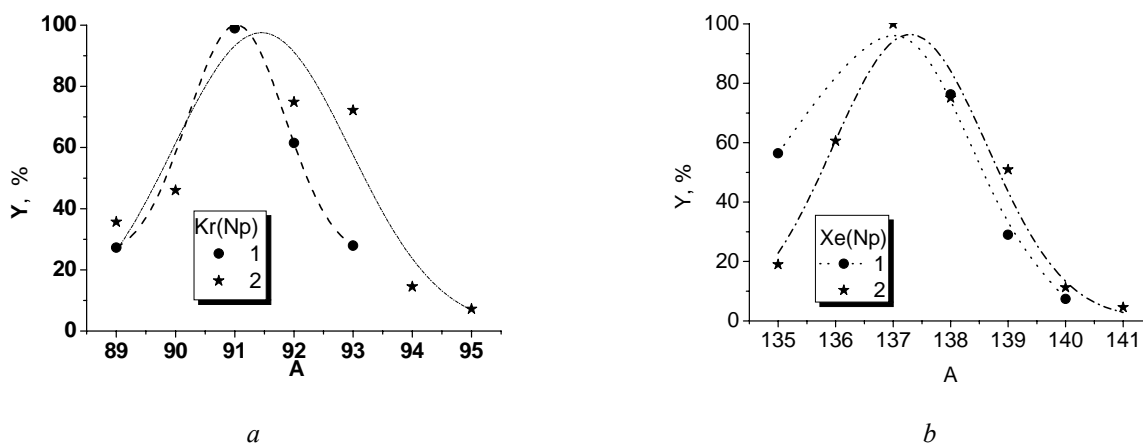


Fig. 4. Comparison of the experimental and theoretical mass spectra of the Kr (a) and Xe (b) isotopes at the  $^{237}\text{Np}$  fission.

### 3. Conclusion

Thus, the results presented indicate, in general, fairly good capabilities of suggested theory in interpreting the experiments on the heavy nuclei MCSFF. Improvement of reliability of the above comparison requires more exact binding energy values for atomic nuclei and dictates the necessity to take into account the statistical fluctuations of thermodynamical parameters for small systems, which the atomic nuclei belong to.

The authors are grateful to Yu. Kibkalo and M. Stetc for fruitful discussions and O. Snigursky for technical support. This work was carried out with the partial support under the Project K-237 of the State Program of Fundamental and Applied Research.

### REFERENCES

1. *Oganesian Yu.Ts., Lazarev Yu.A.* Heavy ions and nuclear fission. Treatise on heavy ion science / Ed. by D. A. Bromly. - N.Y.: Plenum Press, 1985. - Vol. 4. - P. 1 - 54.
2. *Zaika N.I., Kibkalo Yu. V., Tokarev V.P.* Actinide nuclei fission at medium excitation energies. *Collective Nuclear Dynamics // The Fourth KINR International School on Nuclear Physics.* - Kiev, 1994. - P. 399 - 415.
3. *Aomann T., Begemann-Blaich M., Benluire J. et al* Investigation of Actinide Nuclei by Fragmentation of  $^{238}\text{U}$  // *Phys. Lett.* - 1994. - Vol. B325. - P. 313 - 319.
4. *Gangskij Yu., Zhemnik V., Maslova N. et al.* Independent Kr и Xe fragments yields under the  $^{237}\text{Np}$  photofission (in Russian). - Dubna, 2003. - 11 p. - (Preprint / JINR; P15-2003-192).
5. *Maslyuk V.* New statistical approach to the systematization of heavy-nuclei fission fragmenta // *Intern. J. of Physics.* - 2000. - Vol. 6, No. 1 - 2. - P. 1.
6. *Facchini U., Pierini G., Saetta-Menichella E.* Statistical aspects in fission mechanism of actinides// *Energia Nucleare.* - 1973. - Vol. 20, No. 12. - P. 667 - 677.
7. *Chbihi A., Schapiro O., Solou S., Gross D. H.E.* Nuclear shell effects at high temperature// *Europ. Phys. J.* - 1999. - Vol. A5. - P. 251.
8. *Moller P., Nix J.P., Myers W.D., Swiatecky W.J.* *Atomic Data and Nuclear Data Tables.* - 1995. - Vol. 59. - P. 185.

## **НОВАЯ ЦВЕТНАЯ СТАТИСТИКА ДЛЯ СИСТЕМАТИЗАЦИИ ОСКОЛКОВ ДЕЛЕНИЯ ТЯЖЕЛЫХ ЯДЕР**

**В. Т. Маслюк, О. А. Парлаг, А. І. Лендел, Т. Й. Маринец**

Предложен новый статистический метод для систематизации осколков деления тяжелых ядер и изучения их массовых, а также зарядовых спектров, следующий из исследования условий стойкости постделительного ансамбля кластеров ядер-осколков. При этом протоны (нейтроны) разных осколков рассматриваются как статистически неэквивалентные. Показана возможность применения новой статистики для объяснения известных экспериментов по делению тяжелых ядер.

## **НОВА КОЛЬОРОВА СТАТИСТИКА ДЛЯ СИСТЕМАТИЗАЦІЇ УЛАМКІВ ПОДІЛУ ВАЖКИХ ЯДЕР**

**В. Т. Маслюк, О. О. Парлаг, О. І. Лендел, Т. Й. Маринець**

Запропоновано новий статистичний метод для систематизації уламків поділу важких ядер та дослідження їх масових та зарядових спектрів, який впливає з дослідження умов стійкості постподільного ансамблю кластерів ядер-уламків. При цьому протони (нейтрони) різних уламків розглядаються як статистично нееквівалентні. Показано можливість застосування нової статистики для пояснення відомих експериментів з поділу важких ядер.