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CONTRIBUTION OF THE EXCITED 1^+ STATES TO THE ^{116}Cd 2ν2β-TRANSITION AMPLITUDE

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The half-life and differential intensities of 2ν2β-transitions in ^{116}Cd have been calculated with the account for the contributions of two first excited 1^+ states of intermediate nucleus.

The expression for 2ν2β-transition amplitude, a second order process in perturbation theory, contains summation over all virtual intermediate nuclear states [1 - 4]. Thus, the set of values of nuclear matrix elements is necessary for the calculations of 2ν2β-decay characteristics, and this is a rather complex problem due to certain difficulties, the existing nuclear models experience in description of all states of intermediate nucleus. The single state dominance hypothesis (SSD) [5, 6] suggests, that the amplitude of 2ν2β-transition, where the ground state of intermediate nucleus is 1^+ state is determined by two step transition, which connects initial and final states through this 1^+-ground state of intermediate nucleus. Then the inverse half-life can be expressed in the following form:

$$\left[t_{1/2}^{2\nu2\beta}\left(0^+\to J_f^\pi\right)\right]^{-1}=\frac{G_\beta^4 g_A^4}{32\pi^7\ln 2}\times$$

$$\times\left|M_1^I M_1^F\right|^2\int\limits_{m_e}^{T+m_e}d\varepsilon_1\int\limits_{m_e}^{T+2m_e-\varepsilon_1}d\varepsilon_2\int\limits_{0}^{T+2m_e-\varepsilon_1-\varepsilon_2}d\omega_1\times \tag{1}$$

$$\times F(Z_f,\varepsilon_1)F(Z_f,\varepsilon_2)p_1\varepsilon_1 p_2\varepsilon_2\omega_1^2\omega_2^2 D(K,L),$$

where p_1, p_2 and ε_1, ε_2 are respectively momenta and energies of electrons, ω_1, ω_2 are energies of antineutrinos, $\omega_2 = T + 2m_e - \varepsilon_1 - \varepsilon_2 - \omega_1$, $T = E_i - E_f - 2 = Q_{\beta\beta}$ is the total kinetic energy of leptons in final state, and $E_i(E_f)$ is the mass of parent (daughter) nucleus. $F(Z_f, \varepsilon)$ is the relativistic Coulomb factor.

$$D(K,L)=\begin{cases}(K^2+L^2+KL)/3 & \text{if } J_f^\pi = 0^+\\ (K-L)^2/3 & \text{if } J_f^\pi = 2^+\end{cases} \tag{2}$$

The factors K and L in (2) are built of energy denominators of second order perturbation theory as follows:

$$\mathrm{K}=\frac{1}{\mu_1+\left(\varepsilon_1+\omega_1-\varepsilon_2-\omega_2\right)/2}+\frac{1}{\mu_1-\left(\varepsilon_1+\omega_1-\varepsilon_2-\omega_2\right)/2}$$

$$\mathrm{L}=\frac{1}{\mu_1+\left(\varepsilon_1+\omega_2-\varepsilon_2-\omega_1\right)/2}+\frac{1}{\mu_1-\left(\varepsilon_1+\omega_2-\varepsilon_2-\omega_1\right)/2}$$

with $\mu_1 = E_{1^+_{g.s.}} - (E_i + E_f)/2,$, $E_{1^+_{g.s.}}$ is the energy of the 1^+-ground state of the intermediate nucleus.

Nuclear matrix elements $M_1^I = \langle 1^+_{g.s.} \| \hat{\beta}^- \| 0_i \rangle$ and $M_F^I = \langle J_f^\pi \| \hat{\beta}^- \| 1^+_{g.s.} \rangle$ can be calculated theoretically [7] or deduced from log ft values of electron capture and single beta decay processes.

If log ft_{EC} and log ft_{β^-} are used, then

$$M_1^I = \frac{1}{g_A}\sqrt{\frac{3D}{ft_{EC}}}, \quad M_1^F = \frac{1}{g_A}\sqrt{\frac{3D}{ft_{\beta^-}}}, \text{ where } D = \frac{2\pi^3 \ln 2}{G_\beta^2 m_e^5} = 6293{,}04 \text{ s. } G_\beta = G_F \cos\theta_C,$$

$G_F = 1{,}16639 \cdot 10^{-5} / \text{GeV}^2$, $\cos\,\theta_C = 0{,}9738$. [8].

The following approximation is usually employed in estimations of 2ν2β half-life – it is supposed, that outgoing leptons have approximately equal values of kinetic energy [1 - 3]. Then $K \approx L$ and $D(K, L) \approx 4/\mu^2$ for $J_f^\pi = 0^+$. As it was shown in [9 - 11], the approach within SSD, when the dependence of denominators of perturbation theory on lepton energies is ignored, can lead to appreciable overestimation of 2ν2β half decay-time. In case of $0^+ \rightarrow 0^+_{g.s.}$ transitions in ^{100}Mo this effect is about 25 %.

Within single state dominance hypothesis the single electron energy and angular distribution differential decay rates for 2ν2β-transition to 0^+ final state have the following form:

$$\frac{d\omega(0^+ \rightarrow 0^+)}{d\varepsilon_1} = \left(\frac{G_\beta^4 g_A^4}{32\pi^7}\right) \left|M_1^I M_1^F\right|^2 p_1\varepsilon_1 F(Z_f,\varepsilon_1) \int_{m_e}^{E_i - E_f - \varepsilon_1} p_2\varepsilon_2 F(Z_f,\varepsilon_2) d\varepsilon_2 \times$$

$$\times \int_0^{E_i - E_f - \varepsilon_1 - \varepsilon_2} \omega_1^2 \omega_2^2 \frac{K^2 + KL + L^2}{3} d\omega_1 \qquad (2)$$

$$\frac{d\omega(0^+ \rightarrow 0^+)}{d\cos\theta} = \left(\frac{G_\beta^4 g_A^4}{64\pi^7}\right) \left|M_1^I M_1^F\right|^2 \int_{m_e}^{E_i - E_f - m_e} p_1\varepsilon_1 F(Z_f,\varepsilon_1) d\varepsilon_1 \int_{m_e}^{E_i - E_f - \varepsilon_1} p_2\varepsilon_2 F(Z_f,\varepsilon_2) \times$$

$$\times \int_0^{E_i - E_f - \varepsilon_1 - \varepsilon_2} \omega_1^2 \omega_2^2 \left(\frac{K^2 + KL + L^2}{3} + \frac{p_1 p_2}{\varepsilon_1 \varepsilon_2} \frac{2K^2 + 5KL + 2L^2}{9} \cos\theta\right) d\omega_1 . \qquad (3)$$

Here for the sake of simplicity we do not take into consideration correction factors $R_{11}^-(\varepsilon_1), R_{11}^-(\varepsilon_2)$, which are rather close to unit and diminish half decay time values for ^{100}Mo, ^{116}Cd on few percents [12]. The expressions for $F(Z_f, \varepsilon)$, $R_{11}^-(\varepsilon)$ are given in [1, 13].

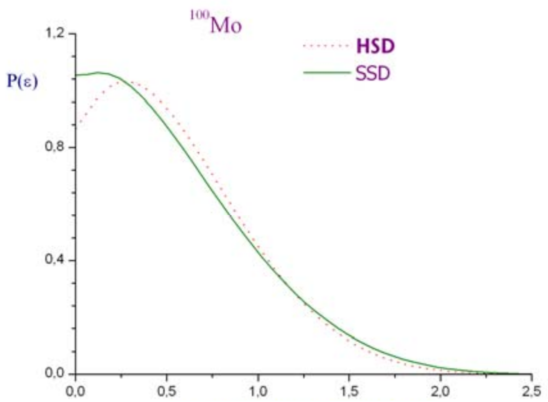

Fig. 1. Electron kinetic energy.

The effect of energy denominators might influence strongly the differential rates. The use of standard approximation for K and L factors, i.e. if they do not depend on lepton energies, corresponds to the case, when transitions through the higher lying states of intermediate nucleus give the main contribution to the 2ν2β-transition amplitude. This situation is equivalent to the higher state dominance (HSD) hypothesis [11, 12].

The relation (1) gives the value of $t_{1/2}^{2\nu 2\beta}$ for ^{100}Mo 2ν2β transition to the ground state of ^{100}Ru $6{,}8 \cdot 10^{18}$ years, which coincides with experimental data, obtained recently by NEMO-3 experiment [14]. Single electron decay rates $P(\varepsilon) = 1/\omega\, d\omega/d\varepsilon$ for ^{100}Mo isotope corresponding to SSD and HSD hypothesis are shown in Fig. 1.

It should be noted, that in the considered approach the differential decay rates, normalized to the full decay rate are free of the nuclear matrix elements. NEMO-3 is a unique setup giving the possibility to measure energy distribution of a single electron and angular correlation of outgoing electrons. The experimental results, obtained for $P(\varepsilon)$ are in favor of SSD nuclear mechanism of two-neutrino double beta-decay in ^{100}Mo [14].

The angular correlation curve for outgoing electrons, as it follows from (3) is a straight line

$$\frac{d\omega(0^+ \to 0^+)}{d\cos\theta} = \frac{1}{2}\omega(0^+ \to 0^+)[1+\kappa\cos\theta]. \tag{4}$$

For ^{100}Mo κ = -0,627 (SSD), κ = -0,.646 (HSD).

The situation is more complicated for two-neutrino beta decay of ^{116}Cd. SSD calculations [9 - 12] give $t_{1/2}^{2\nu 2\beta}$ = 1,1 · 10^{19} years for this isotope, whereas the measured value, obtained in two experiments NEMO-3 [14] and INR-INFN, Solotvina, Ukraine [15] on the base of different methods is more than twice larger: (2,8 ± 0,3)·10^{19} years. So it is reasonable in this case to take into account contribution of higher excited states in 2ν2β-amplitude. Thus, one should use general expressions for $t_{1/2}^{2\nu 2\beta}$ and differential decay rates [4]

$$\left[t_{1/2}^{2\nu 2\beta}\left(0^+ \to 0_f^+\right)\right]^{-1} = \frac{G_\beta^4 g_A^4}{32\pi^7 \ln 2} \times$$

$$\times \int_{m_e}^{T+m_e} de_1 \int_{m_e}^{T+2m_e-\varepsilon_1} d\varepsilon_2 \int_0^{T+2m_e-\varepsilon_1-\varepsilon_2} d\omega_1 \times$$

$$\times F(Z_f,\varepsilon_1)F(Z_f,\varepsilon_2)p_1\varepsilon_1 p_2\varepsilon_2\omega_1^2\omega_2^2 A_{0_f^+}$$

$$\frac{d\omega(0^+ \to 0^+)}{d\varepsilon_1} = \left(\frac{G_\beta^4 g_A^4}{32\pi^7}\right) p_1\varepsilon_1 F(Z_f,\varepsilon_1) \int_{m_e}^{E_i-E_f-\varepsilon_1} p_2\varepsilon_2 F(Z_f,\varepsilon_2)d\varepsilon_2 \int_0^{E_i-E_f-\varepsilon_1} \omega_1^2\omega_2^2 A_{0_f^+}$$

$$\frac{d\omega(0^+ \to 0^+)}{d\cos\theta} = \left(\frac{G_\beta^4 g_A^4}{64\pi^7}\right) \int_{m_e}^{E_i-E_f-m_e} p_1\varepsilon_1 F(Z_f,\varepsilon_1)d\varepsilon_1 \int_{m_e}^{E_i-E_f-\varepsilon_1} p_2\varepsilon_2 F(Z_f,\varepsilon_2)$$

$$\times \int_0^{E_i-E_f-\varepsilon_1-\varepsilon_2} \omega_1^2\omega_2^2 (A_{0_f^+} + \frac{p_1 p_2}{\varepsilon_1\varepsilon_2} B_{0_f^+}\cos\theta)d\omega_1,$$

where

$$4A_{0_f^+} = \left|\sum_N \left\langle 0_F^+ \left\|\tau^+\sigma\right\| 1_N^+ \right\rangle \left\langle 1_N^+ \left\|\tau^+\sigma\right\| 0_I^+ \right\rangle (K_N + L_N)\right|^2 + \frac{1}{3}\left|\sum_N \left\langle 0_F^+ \left\|\tau^+\sigma\right\| 1_N^+ \right\rangle \left\langle 1_N^+ \left\|\tau^+\sigma\right\| 0_I^+ \right\rangle (K_N - L_N)\right|^2 \tag{5}$$

$$4B_{0_f^+} = \left|\sum_N \left\langle 0_F^+ \left\|\tau^+\sigma\right\| 1_N^+ \right\rangle \left\langle 1_N^+ \left\|\tau^+\sigma\right\| 0_I^+ \right\rangle (K_N + L_N)\right|^2 - \frac{1}{9}\left|\sum_N \left\langle 0_F^+ \left\|\tau^+\sigma\right\| 1_N^+ \right\rangle \left\langle 1_N^+ \left\|\tau^+\sigma\right\| 0_I^+ \right\rangle (K_N - L_N)\right|^2 \tag{6}$$

$$K_N = \frac{1}{\mu_N + (\varepsilon_1 + \omega_1 - \varepsilon_2 - \omega_2)/2} + \\ + \frac{1}{\mu_N - (\varepsilon_1 + \omega_1 - \varepsilon_2 - \omega_2)/2}$$

$$L_N = \frac{1}{\mu_N + (\varepsilon_1 + \omega_2 - \varepsilon_2 - \omega_1)/2} + \\ + \frac{1}{\mu_N - (\varepsilon_1 + \omega_2 - \varepsilon_2 - \omega_1)/2}$$

with $\mu_N = E_N - (E_i + E_f)/2$. The experimental log *ft* values can give only absolute values of nuclear matrix elements $\langle 1_N^+ \| \tau^+\sigma \| 0_I^+ \rangle$, $\langle 0_F^+ \| \tau^+\sigma \| 1_N^+ \rangle$ for N = 1, i.e. for the ground state of intermediate nucleus. As it can be seen from (4), (5), nuclear matrix elements, corresponding to the total set of excited states of intermediate nucleus are necessary for estimation of 2ν2β characteristics. The calculation of these quantities, both absolute and relative phases, is a rather difficult theoretical problem.

The investigation of charge-exchange reactions gives the possibility to obtain GT^-, GT^+ strength distributions for transitions of interest and deduce absolute values of nuclear matrix elements for low-lying excitations of intermediate nucleus. From the results on $^{116}Cd(^3He, t)^{116}In$ [16] and $^{116}Sn(d, ^2He)$ ^{116}In [17] reactions experiments the absolute values of nuclear matrix elements for two first 1^+ excited states of ^{116}In, which is intermediate nucleus for ^{116}Cd 2β-transitions can be found. They are presented in the Table together with nuclear matrix element, which corresponds to the ground state of ^{116}In, derived from log *ft* data.

$E_N(1^+) - E_{g.s.}$, MeV	$\lvert\langle 1_N^+ \| \tau^+\sigma \| 0_I^+ \rangle\rvert$	$\lvert\langle 0_F^+ \| \tau^+\sigma \| 1_N^+ \rangle\rvert$
0.	0,699	0,511
1,00	0,346	0,332
2,2	0,412	< 0,265

Here we suppose, that in addition to the ground state of ^{116}In, the contribution of these two first excited 1^+states should be taken into account in (5), (6) for ^{116}Cd 2ν2β calculations. Moreover we suppose that their contributions are of opposite signs in order to obtain the decrease of theoretical decay intensity compared with the pure SSD estimations. In general case complex phases of matrix elements should be taken into consideration. For 2,2 MeV excitation energy we use the maximal possible value of $\lvert\langle 0_F^+ \| \tau^+\sigma \| 1_N^+ \rangle\rvert$, which equals 0,265. So for $t_{1/2}^{2\nu 2\beta}$, we obtain the value of $2{,}57 \cdot 10^{19}$ years, which coincides with experimental data.

The calculations, produced in [17] lead to value of $4{\cdot}10^{19}$ years for ^{116}Cd $t_{1/2}^{2\nu 2\beta}$. The reason of the discrepancy between this result and our approach is the following. In [17] the half-life has been obtained in the approximation of equal kinetic energies of outgoing leptons according to the formula

$$\left[t_{1/2}^{2\nu 2\beta}\right]^{-1} = G^{(2\nu)} \left| M^{(2\nu)}(DGT) \right|^2 ,$$

where $G^{(2\nu)}$ is the phase factor and the double Gamow - Teller matrix element is determined as

$$M^{(2\nu)}(DGT) = \sum_N \frac{\langle 0_{g.s}^{(f)} \| \tau^+\sigma \| 1_N^+ \rangle \langle 1_N^+ \| \tau^+\sigma \| 0_{g.s.}^{(i)} \rangle}{\mu_N} = \sum_N \frac{M_N(GT^+) M_N(GT^-)}{\mu_N} = \sum_N M_N(DGT) . \quad (7)$$

Here $\mu_N = E(1_N^+) - (E_i + E_f)/2$ and $E(1_N^+)$ is the energy of the N-th intermediate 1^+ state. For M_1^I the result of [16], obtained from (^3He, *t*) charge exchange reaction was used in [17], M_1^I = 0,178, which deviates strongly from matrix elements, estimated from log ft_{EC} data, M_1^I = 0,699, which we incorporate in our

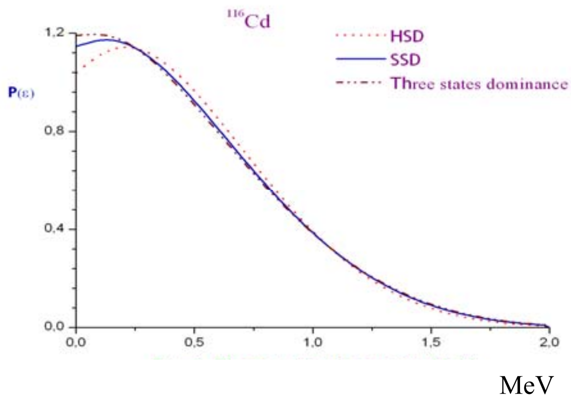

Fig. 2. Electron kinetic energy.

calculations. So in order to reproduce experimental half-life, it has been assumed in [17], that matrix elements $M_N(DGT)$ in (7) add constructively. Whereas in our approach the energy denominators of perturbation theory are free of any approximations and we assume, that contributions of first two excited states have opposite signs. Further measurements, both of charge exchange cross section and differential 2ν2β transition rates are necessary to examine this question.

The single electron decay rates $P(\varepsilon) = 1/\omega d\omega/d\varepsilon$ for ^{116}Cd isotope corresponding to SSD and HSD and three-state dominance hypothesis are built in Fig. 2.

The angular correlation parameter κ (4) have the following values for different approaches for calculation of 2ν2β transition in ^{116}Cd: κ = -0,602 (three states dominance); κ = -0,606 (SSD); κ = = -0,615 (HSD). These results can be compared with precise high statistics measurements with ^{116}Cd, which are now in progress in NEMO-3 experiment, in order to test different hypothesis on nuclear mechanism of 2ν2β transition in this isotope.

In summary we note, that SSD hypothesis should be generalized in the case of ^{116}Cd. The precise experimental data on differential rates, compared with theoretical distributions can give interesting information on nuclear mechanism of double beta decay and for construction of nuclear structure models. The further measurements of charge exchange reactions for a number of isotope sources are valuable for double beta decay investigations. It would be constructive to apply the performed approach to ^{48}Ca, the corresponding charge exchange reactions for this isotopes are investigated in [18].

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ВКЛАД ВОЗБУЖДЕННЫХ 1^+ СОСТОЯНИЙ В АМПЛИТУДУ 2ν2β РАСПАДА ^{116}Cd

С. В. Семенов, Ю. В. Гапонов, Ф. Шимковиц, Р. Дворницкий

Проведен расчет периода полураспада и дифференциальных интенсивностей в 2ν2β распаде ^{116}Cd с учетом двух первых возбужденных уровней промежуточного ядра.

ВНЕСОК ЗБУДЖЕНИХ 1^+ СТАНІВ В АМПЛІТУДУ 2ν2β РОЗПАДУ ^{116}Cd

С. В. Семенов, Ю. В. Гапонов, Ф. Шимковіц, Р. Дворницький

Проведено розрахунок періоду напіврозпаду та диференційних інтенсивностей в 2ν2β розпаді ^{116}Cd з урахуванням двох перших збуджених станів проміжного ядра.