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Magic Numbers of Ultraheavy Nuclei

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Abstract—For nuclei where the number of protons lies in the range $76 \leq Z \leq 400$, proton and neutron shell corrections are calculated along the beta-stability line described by Green's formula. The magic numbers of protons and neutrons are determined for ultraheavy nuclei. Alpha-decay half-lives and fission barriers are estimated for ultraheavy doubly magic nuclei. © *2005 Pleiades Publishing, Inc.*

INTRODUCTION

Magic numbers corresponding to the filling of nuclear shells of single-particle levels have been known since the middle of the past century, and the role that they play in nuclear physics is of crucial importance [1–4]. Nuclei where the numbers of nucleons are magic are more stable and have a higher binding energy than their neighbors and are more abundant in nature than them [1–4]. Many quantities, such as the energies required for the separation of one and two nucleons, the energies of alpha and beta transitions, pairing energies, and the excitation energies of low-lying vibrational states, undergo discontinuities upon passing a magic number [1–4].

The magic numbers $Z = 82$ and $N = 126$ are the greatest magic numbers that have been empirically confirmed to date for protons and neutrons, respectively. It should be noted, however, that, at matter densities close to nuclear-matter densities, there occurs a transition in neutron stars from nuclei, neutron drops, neutrons, and protons to fusing and decaying heavy nuclei [5, 6], which transform, as the density increases further, into a more complicated state of nuclear matter [5–7]. Therefore, very heavy nuclei can be formed in neutron stars. The existence of neutron-rich nuclei where the number of neutrons is about 10^3–10^5 and of supercharged nuclei where the number of protons is about 1600 is discussed in [6]. It would be interesting to find magic numbers in ultraheavy nuclei where the number of nucleons falls within the range $300 \leq A \leq 1200$. In neutron stars, the relative production rate for ultraheavy nuclei involving a magic number of nucleons would be enhanced because of their higher stability. The results presented in [8] also provoke interest in studying magic numbers in the region of superheavy nuclei.

It is well known that magic numbers correspond to the filling of nucleon shells in spherical beta-stable nuclei [1–4, 9–12]. The shell correction has a deep local minimum in the vicinity of a magic number [4, 9–14]. Calculating shell corrections for spherical nuclei involving various numbers of protons and neutrons, one can therefore determine magic numbers from the positions of deep local minima in the proton and neutron shell corrections. It should be noted that nuclei lying along the beta-stability line and having empirically known magic numbers of nucleons ($Z =$ 8, 20, 28, 50, 82, $N =$ 8, 20, 28, 50, 82, 126) are spherical [4, 11]. In the following, we will therefore also explore shell corrections in spherical nuclei.

CALCULATION OF SHELL CORRECTIONS

Figure 1 shows the proton (δ_P), neutron (δ_N), and total ($\delta_P + \delta_N$) shell corrections calculated for $76 \leq Z \leq 400$ even–even spherical nuclei lying along the beta-stability line approximated by Green's formula [15], from which it follows that a nucleus involving Z protons and $N_{\text{Green}}(Z) = (2/3)Z + (5/3) \times (10\,000 + 40Z + Z^2)^2 - 500/3$ neutrons corresponds to the beta-stability valley [15]. Green's formula describes well the beta-stability line, which is associated with a specific relation between the numbers of protons and neutrons for nuclei known to date. Let us assume that that the relation between the numbers of protons and neutrons in beta-stable nuclei that is described by Green's formula is valid for heavier nuclei inclusive. Figure 1 shows the shell corrections calculated for nuclei involving an even number Z of protons in the range from 76 to 400 and an even number N of neutrons in the range $N_{\text{Green}} - 10 \leq N \leq N_{\text{Green}} + 10$, where N_{Green} is the even number closest to $N_{\text{Green}}(Z)$. The numbers of neutrons and nucleons in nuclei were varied in the range $102 \leq N \leq 820$ and in the range $178 \leq A \leq 1218$, respectively.

The energies of single-particle levels of nucleons were calculated for the nucleon mean field in the form of the Woods–Saxon potential with allowance

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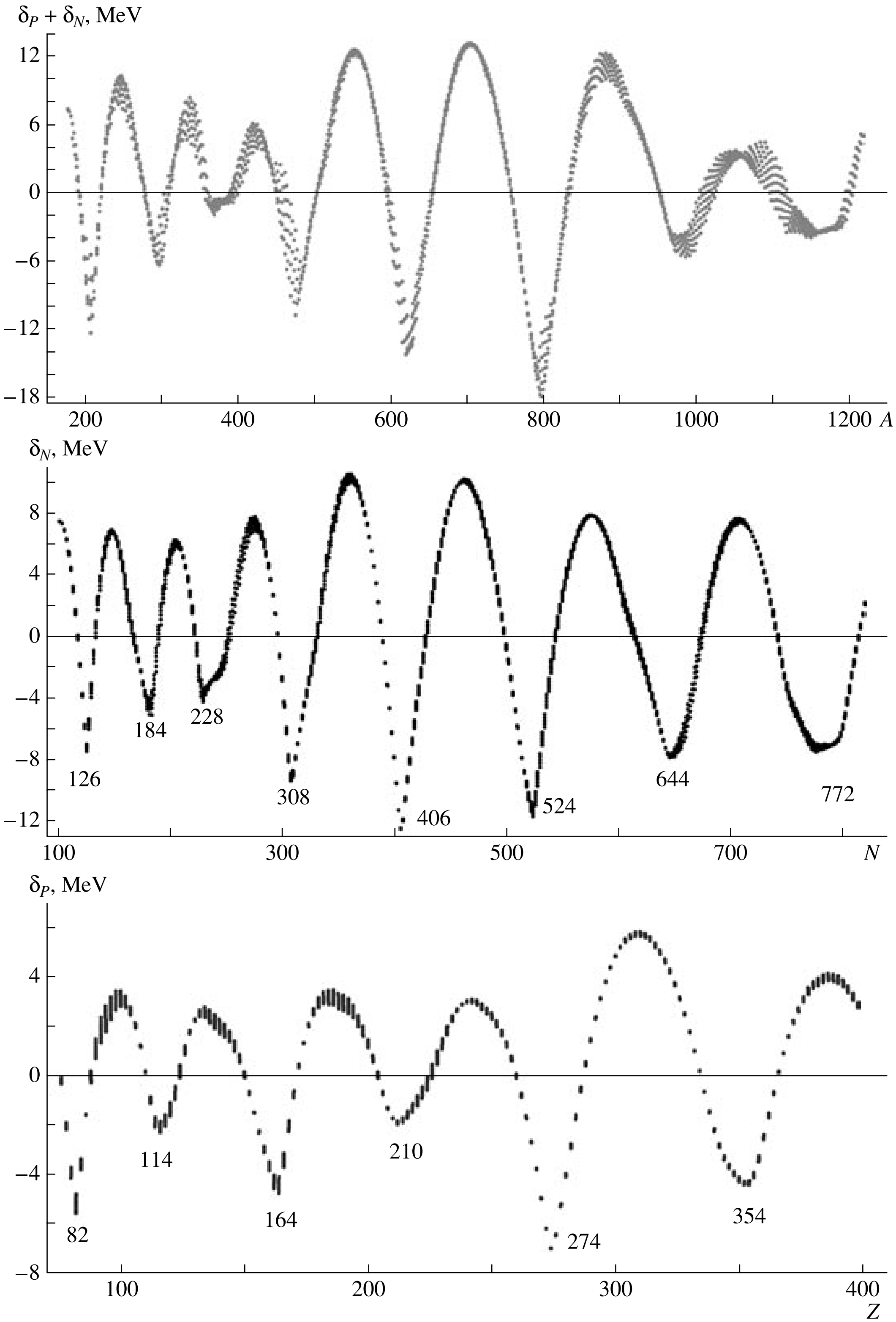

Fig. 1. Proton (δ_P), neutron (δ_N) and total ($\delta_P + \delta_N$) shell corrections for even–even spherical nuclei.

for spin–orbit and Coulomb interactions [2–4, 11, 16]. We employed a "universal" set of parameters of the Woods–Saxon potential [16]. This set makes it possible to describe well the spectra of single-particle levels in light, medium-mass, heavy, and superheavy spherical and deformed nuclei. Also, it was successfully used to calculate various properties of nuclei [14, 16, 17]. The residual pairing interaction of nucleons was taken into account in the Lipkin–Nogami approximation [13, 18] , the coupling constant being set to $r_{\rm mic} = 3.30$ fm [13]. In order to calculate shell corrections, we employed a basis formed by oscillator wave functions for the lowest 35 shells of an axially deformed harmonic oscillator and took into account the lowest 2330 single-particle levels. The degree of the correcting polynomial was chosen to be six. This choice is conventional in calculating shell corrections for medium-mass, heavy, and superheavy nuclei [11]. The energies of single-particle levels of nucleons were calculated with the aid of the WSBETA code [16], which was refined in order to take into account greater numbers of shells and levels.

From the data in Fig. 1, it follows that the proton shell corrections have deep local minima at $Z =$ 82, 114, and 164 and that the neutron shell corrections have deep local minima at $N = 126$, 184, and 228. It should be noted that $Z = 82$, 114, and 164 and $N = 126$, 184, and 228 are or are assumed to be magic numbers [4]. For example, the empirically known magic numbers $Z = 82$ and $N = 126$ correspond to the doubly magic spherical nucleus $^{208}\mathrm{Pb}_{126}$. The values that we found for the shell corrections in the doubly magic nuclei $^{208}\mathrm{Pb}_{126}$ and $^{298}114_{184}$ are in good agreement with their counterparts calculated in [13, 14]. Thus, our calculations reproduce known results and make it possible to perform an extrapolation to the region of heavier nuclei.

The proton shell corrections have three deep local minima in the region $164 < Z \leq 400$ (Fig. 1). Therefore, $Z = 210$, 274, and 354 are the magic numbers of protons in this region. Analyzing deep local minima in the dependence of the neutron shell corrections on the number of neutrons in Fig. 1 in the range $228 < N \leq 820$, we can conclude that $N = 308$, 406, 524, 644, and 772 are the magic numbers of neutrons in this region.

ALPHA-DECAY HALF-LIVES

Among nuclei for which we have calculated shell corrections and which are close to the beta-stability line described by Green's formula, the $^{208}\mathrm{Pb}_{126}$, $^{298}114_{184}$, $^{472}164_{308}$, $^{616}210_{406}$, and $^{798}274_{524}$ nuclei are doubly magic. In Fig. 1, the total shell correction $\delta_P + \delta_N$ has deep local minima in the vicinities of these doubly magic nuclei. However, the heavier doubly magic nuclei $^{998}354_{644}$ and $^{1126}354_{772}$ are quite far off the beta-stability line described by Green's formula. For example, the $^{998}354_{644}$ nucleus is neutron-deficient, while the $^{1126}354_{772}$ nucleus is neutron-rich.

Alpha decay and fission are the main modes of decay of doubly magic beta-stable heavy nuclei. Let us estimate the alpha-decay half-lives and fission barriers for the super- and ultraheavy nuclei $^{298}114_{184}$, $^{472}164_{308}$, $^{616}210_{406}$, and $^{798}274_{524}$.

The alpha-decay half-lives for these nuclei will be found with the aid of the phenomenological Viola–Seaborg formula [19], which relates the alpha-decay half-life to the energy of alpha particles and the charge of the primary nucleus. For the constants of the phenomenological Viola–Seaborg formula, the authors of [20] found values that made it possible to reproduce faithfully the experimental half-lives of 58 nuclei heavier than $^{208}\mathrm{Pb}_{126}$. Knowing the total shell corrections calculated here and the macroscopic binding energies of nuclei as calculated by means of the mass formula from [13], we determine the energies of alpha particles (in MeV) emitted by the aforementioned super- and ultraheavy nuclei. We have

$$Q(^{298}114_{184}) \approx 9.4, \quad Q(^{472}164_{308}) \approx 13.1,$$
$$Q(^{616}210_{406}) \approx 20.9, \quad Q(^{798}274_{524}) \approx 35.0.$$

After that, we determine the half-lives with respect to the alpha decay of these nuclei with the aid of the modified Viola–Seaborg formula [20]. The results are (in s)

$$T_{1/2}(^{298}114_{184}) \approx 1.1 \times 10^{2},$$
$$T_{1/2}(^{472}164_{308}) \approx 2.3 \times 10^{4},$$
$$T_{1/2}(^{616}210_{406}) \approx 4.2 \times 10^{-6},$$
$$T_{1/2}(^{798}274_{524}) \approx 3 \times 10^{-21}.$$

The half-lives of the first three doubly magic nuclei with respect to alpha decay are quite long and can readily be measured. The results that we obtained for the energy of alpha particles from the $^{298}114_{184}$ nucleus and for its alpha-decay half-life are in good agreement with the results reported in [14].

Let us estimate fission barriers in doubly magic ultraheavy nuclei. In order to do this, we calculate the deformation energy of nuclei that is associated with the change in their shape. For very heavy nuclei, the fission barrier can be roughly estimated by taking into account only variations in the quadrupole deformation β_2 of the nuclear surface, since the quantity β_2 is small at the barrier, with the result that deformations of higher multipole orders have a weaker effect on the barrier shape. Figure 2 shows the deformation energy as a function of β_2 for the super- and ultraheavy doubly magic nuclei $^{298}114_{184}$, $^{472}164_{308}$, $^{616}210_{406}$, and $^{798}274_{524}$. In calculating nuclear deformation energies, we took into account variations in the liquid-drop and shell energies. The liquid-drop energy was found in the approximation where nuclear interaction is represented as the sum of the Yukawa and exponential terms [21].

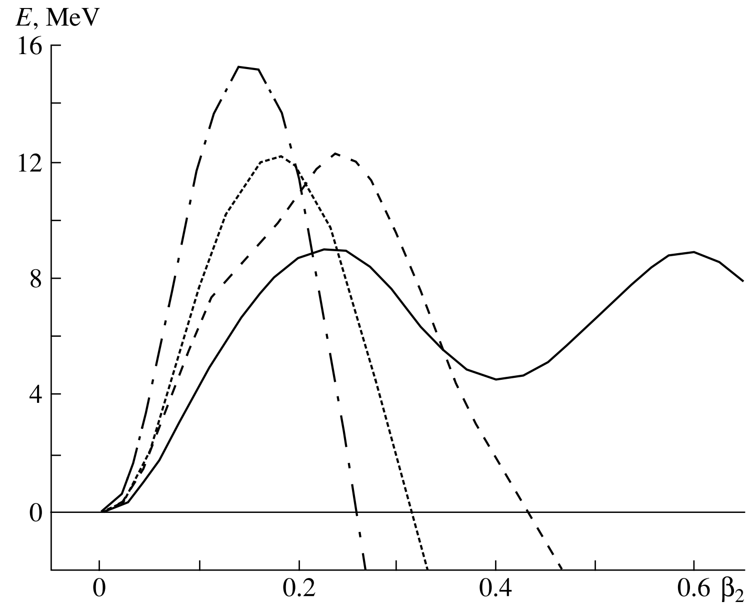

Fig. 2. Deformation energy for the super- and ultraheavy doubly magic nuclei (solid curve) $^{298}114_{184}$, (dashed curve) $^{472}164_{308}$, (dotted curve) $^{616}210_{406}$, and (dash-dotted curve) $^{798}274_{524}$.

From the data in Fig. 2, it follows that the fission barrier in the $^{298}114_{184}$ nucleus has a two-humped shape. The identical fission-barrier shape was obtained in [14] for the $^{298}114_{184}$ nucleus. The inner- and outer-barrier heights determined from the data in Fig. 2 are larger than their counterparts calculated in [14], this being due to the fact that the multipole surface deformations β_2, β_3, β_4, ..., β_8 were taken into account in [14]. However, the inner-fission-barrier heights determined from Fig. 2 and in [14] are rather close, because, at small values of β_2, the effect of deformations of higher multipole orders on the barrier height is weak. This makes it possible to estimate the shape and the height of fission barriers in ultraheavy doubly magic nuclei.

From Fig. 2, one can see that the ultraheavy doubly magic nuclei $^{472}164_{308}$, $^{616}210_{406}$, and $^{798}274_{524}$ have rather high but narrow one-humped fission barriers. With increasing nuclear charge, the fission barrier becomes narrower and occurs at smaller values of β_2. An increase in the barrier height with increasing mass number in these doubly magic nuclei is due to an increase in the amplitude of the shell correction (see Fig. 1).

The fission half-life of the $^{298}114_{184}$ nucleus is about 10^{10} times longer than its alpha-decay half-life [14]. The lifetimes of the $114 \leq Z \leq 120$ neighboring nuclei are also determined by their alpha-decay periods [14, 17]. Similarly, the lifetimes of the $^{472}164_{308}$, $^{616}210_{406}$, and $^{798}274_{524}$ nuclei are related to their alpha-decay half-lives, since the fission barriers in these nuclei are rather high. As was indicated above, the half-lives of the doubly magic nuclei $^{472}164_{308}$ and $^{616}210_{406}$ could therefore readily be measured if they were formed.

The nucleus involving the magic number $Z = 164$ of protons can be formed, for example, in the fusion of two lead nuclei, whereas the nucleus involving the magic number $N = 308$ of neutrons can be generated in the fusion of two $^{252}Cf_{154}$ nuclei. However, nuclei arising in these reactions are rather far off the beta-stability line. Ultraheavy doubly magic nuclei can be produced in a collision of two heavy neutron-rich nuclei that is accompanied by the absorption of many neutrons.

DISCUSSION OF THE RESULTS AND CONCLUSION

Recently, there appeared the article of Zhang *et al.* [22], who studied magic numbers for $100 \leq Z \leq 140$ nuclei within the relativistic continuum Hartree–Bogolyubov approximation, employing various versions of microscopic forces. Various nuclear shapes were also taken into account in that article. It is well known that the inclusion of nuclear deformations leads to the emergence of additional local minima in the dependence of shell corrections on the number of nucleons [9–11, 14]. These minima, which are associated with the filling of shells in deformed nuclei, correspond to "quasimagic" numbers. It should be noted that deformed nuclei having filled shells—that is, quasimagic numbers of nucleons—also possess enhanced stability and other properties inherent in spherical beta-stable nuclei involving magic numbers of nucleons. However, the amplitude of magicity effects in deformed nuclei is less than that in spherical nuclei. In the region $100 \leq Z \leq 140$, Zhang *et al.* [22] found a greater number of magic numbers than the present author and the authors of [4, 12]. Some of the magic numbers found in [22] are quasimagic. However, the magic numbers $Z = 114$, $N = 184$, and $N = 228$ for $100 \leq Z \leq 140$ nuclei were found here and in [4, 22] as well.

The shell corrections for $40 \leq Z \leq 200$, $40 \leq N \leq 420$ nuclei were calculated in [12] in the Hartree–Fock approximation with various versions of Skyrme forces and on the basis of the relativistic mean-field model. Within various versions of the calculations, magic numbers in the interval $Z \approx 114$–126, an interval around $Z \approx 164$, the interval $N \approx 172$–184, and an interval around $N \approx 308$ were found in that study. It is worth noting that, in the figures presented in [12], deep minima in the shell corrections are seen in the regions close to $N \approx 228$ and $N \approx 406$; unfortunately these minima are not discussed there.

Because of the Coulomb repulsion of protons, a region depleted in nucleons can arise at the center of super- and ultraheavy nuclei [12, 23]. This effect was taken into account in [12]. However, the doubly magic nucleus $^{472}164_{308}$ found here also proved to be doubly magic in the calculations performed in [12] by using some parametrizations of Skyrme forces. The Coulomb repulsion of protons at the center of a nucleus could affect the values of the magic numbers for heavier nuclei, but a detailed investigation of this effect would require substantially changing the parametrization of the nucleon-mean-field potential. It should be noted that a parametrization that would take into account a decrease in the density at the center of a nucleus has not yet been investigated. Therefore, the effect of the Coulomb repulsion of protons at the center of super- and ultraheavy nuclei was not considered in the present study.

In [12, 22], it was indicated that the values of magic numbers in super- and ultraheavy nuclear regions depend both on the choice of model and on the choice of parameters of forces in microscopic calculations. Therefore, it would be of interest to perform investigations similar to those in [12, 22] for heavier nuclei and to compare the results of such investigation with the results obtained here.

By studying shell corrections, we have determined the proton magic numbers $Z = 114$, 164, 210, 274, and 354 and the neutron magic numbers $N = 184$, 228, 308, 406, 524, 644, and 772 in super- and ultraheavy nuclear regions. Ultraheavy nuclei involving magic numbers of protons and neutrons are expected to be more stable and to possess higher binding energies than neighboring nuclei. The alpha-decay periods of some doubly magic ultraheavy nuclei and fission barriers in them are rather large; therefore, searches for such nuclei are of great interest.

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