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Shell Corrections, Magic Numbers, and Mean Field

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Abstract—It is shown that the positions of deep local minima of shell corrections associated with magic numbers in the region of superheavy nuclei depend on the parameters of the central and spin–orbit mean-field potentials. The accuracy of nuclear-mass predictions made within various models for superheavy nuclei is analyzed.

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1. INTRODUCTION

Advances in experimental investigations recently performed in various laboratories for superheavy nuclei [1–9] have given impetus to theoretically studying various features of superheavy nuclei and mechanisms of their production. The problem of proton and neutron magic numbers in the region of superheavy nuclei is of crucial importance. Magic numbers are associated with the filling of nuclear shells; therefore, nuclei featuring magic numbers of nucleons are more stable than their neighbors. A doubly magic superheavy nucleus may have a long lifetime. A stability island can be formed around such a nucleus. However, experimental data available at the present time [1–11] cannot provide an answer to the question of which numbers are magic for spherical nuclei in the region of superheavy nuclei.

Different nuclear models predict different values of magic numbers for spherical nuclei in the region of superheavy nuclei. For example, parametrizations of the nucleon mean field in the form of the Woods–Saxon potential led to the values of $Z = 114$, 120 and $N = 184$ in [12] and to the values of $Z = 114$ and $N = 184$ in [13–16]. Microscopic self-consistent calculations in the Hartree–Fock–Bogolyubov approximation with Gogny forces yielded $Z = 114$, 120, and 126 and $N = 184$, 164, and 228 [17]. For protons, the magic numbers found on the basis of self-consistent microscopic calculations in the Hartree–Fock–Bogolyubov approximation with Skyrme forces [18] depend on their parametrization and appear to be $Z = 120$ and 124 for the *SkM** parametrization; $Z = 120$ for the *SkI1*, *SkI3*, and *SkI4* parametrizations; $Z = 124$ and 126 for the *SLy4* and *SkP* parametrizations; and $Z = 126$ for the *SkO* parametrization [19–22]. In the case of neutrons, all of these parametrizations lead to the value of $N = 184$ [19–22]. Calculations on the basis of relativistic mean-field models, which were considered in detail in [19–22], yield the magic-number values of $Z = 120$ and $N = 172$ and 184, while the relativistic model proposed in [23] leads to the magic-number values of $Z = 114$ and 120 and $N = 172$, 184, and 258. The values of $Z = 106$, 114, 120, 126, 132, and 138 and $N = 138$, 164, 172, 184, 198, 216, 228, 238, 252, 258, and 274 were determined as the possible magic numbers in performing a detailed analysis of relativistic Hartree–Bogolyubov theory with allowance for a continuum [24], eight versions being considered in parametrizing forces. The semiempirical shell model for nuclear masses that was developed in [25] leads to the magic-number values of $Z = 126$ and $N = 184$ in the region of superheavy nuclei.

Distinctions between theoretical predictions for the proton and neutron magic numbers are quite significant in the region of superheavy nuclei. Therefore, the results obtained by experimentally determining magic numbers for spherical superheavy nuclei could reduce drastically the number of possible models and parametrizations of the mean field or nucleon–nucleon forces. This could improve significantly the accuracy of our knowledge of the structure of superheavy nuclei and the properties of nucleon–nucleon interactions in nuclei. From the theoretical point of view, however, it is necessary to clarify the reasons behind so great a scatter in predictions for magic numbers in going over from one model to another and to find out which parameters in the central or spin–orbit mean field generate variations in magic numbers observed in different models for different mean-field parametrizations.

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It is well known that the shell correction has a deep local minimum in the vicinity of a magic number [12, 26]. Calculating shell corrections for spherical nuclei containing various numbers of protons and neutrons, one can therefore determine magic numbers from the positions of deep local minima in the proton and neutron shell corrections [12, 16, 20, 26]. It follows that, by studying the dependence of shell corrections on the variations in the parameters of the central or spin–orbit mean field, one can find the dependence of magic numbers on the features of the mean field. As will be shown in the next section, the nucleon mean fields differ significantly in different models.

Binding energies and fission barriers are important features of superheavy nuclei. These quantities depend on shell corrections [12, 14–16, 20, 26–29]. For example, the fission barrier in a superheavy nucleus is determined approximately by the shell correction in the ground state [30]. By studying the dependence of the shell correction on variations in the parameters of the central or spin–orbit mean field, one can therefore analyze the dependence of the binding energy and fission barrier on these parameters and the accuracy of our knowledge of these quantities.

This article is organized as follows. In Section 2, we analyze mean-field potentials that are used in various approximations. The dependence of shell corrections and magic numbers on the parameters of the mean-field potential is discussed in detail in Section 3. Section 4 is devoted to analyzing the accuracy in describing masses of superheavy nuclei within various approximations. The conclusions drawn from the present study are given in Section 5.

2. COMPARISON OF NUCLEON MEAN FIELDS IN VARIOUS MODELS

Shell corrections are associated with the deviation of the density of single-particle levels in the vicinity of the Fermi level from the average value of the level density in a given nucleus [12, 26]. If this level density is lower than the average value, the number of states that the system can occupy at low excitations is small. In this case, the shell correction is negative, with the result that the nucleus being considered appears to be more stable to perturbations than its neighbors featuring a higher level density in the vicinity of the Fermi level. It is well known from experimental data that the level density in nuclei involving a magic number of nucleons is lower than that in the neighboring nuclei having a nonmagic number of nucleons [31].

Strutinsky found a relation between the shell correction to the binding energy of the nucleus and the nonuniformity of the single-particle spectrum in the vicinity of the Fermi level [26]. The energies of single-particle levels are the eigenvalues E_ν of the time-independent Schrödinger equation,

$$\left[\frac{\hbar^2}{2m}\nabla + V(r) - E_\nu\right]\psi_\nu(r) = 0, \qquad (1)$$

and are related to the nucleon-mean-field potential $V(r)$. Here, m is the nucleon mass and $\psi_\nu(r)$ is the wave function of the νth state. Since the energies E_ν depend on the potential $V(r)$, the shell corrections and, hence, the positions of deep local minima associated with magic numbers depend on the form of the mean field.

Figure 1 displays the radial dependences of the proton and neutron potentials obtained within various models for the spherical nucleus $^{298}114$. The number of protons, $Z = 114$, and the number of neutrons, $N = 184$, in this nucleus are magic in some models (see Introduction). The mean field in the form of the Woods–Saxon potential involving the universal set of parameters from [27] is widely used to analyze various features of heavy and superheavy elements. The mean-field potentials obtained in the Hartree–Fock approximation for extensively used parametrizations of Skyrme forces (*SkM** [32], *SkP* [33], and *SLy4* [34]) are shown in Fig. 1 along with the radial dependence of the central mean field obtained by folding the Yukawa potential and a stepwise density distribution. This mean field is employed to calculate the shell correction in the mass formulas [14, 28].

The effective nucleon mean-field potentials obtained in the form of the Woods–Saxon potential involving the universal parameters from [27], by means of folding the Yukawa potential and a stepwise density distribution [14], and in the self-consistent Hartree–Fock approximation with *SkP* Skyrme forces are close to one another in the interior of the nucleus, but they differ substantially at its surface (see Fig. 1). The mean-field potentials calculated in the Hartree–Fock approximation with *SkM** and *SLy4* Skyrme forces are much deeper than the remaining potentials. The Woods–Saxon potential whose parameters are set to the values from the universal set has the largest potential-well radius among all of the potentials presented in Fig. 1, the diffuseness-layer thickness for this potential also being the largest. On the contrary, the mean-field potential obtained by folding the Yukawa potential and a stepwise density distribution has the smallest potential-well radius and the smallest diffuseness-layer thickness within the same set of potentials. The radii and diffuseness parameters of the potentials calculated in the Hartree–Fock approximation with Skyrme forces take intermediate values between their counterparts for the Woods–Saxon potential whose

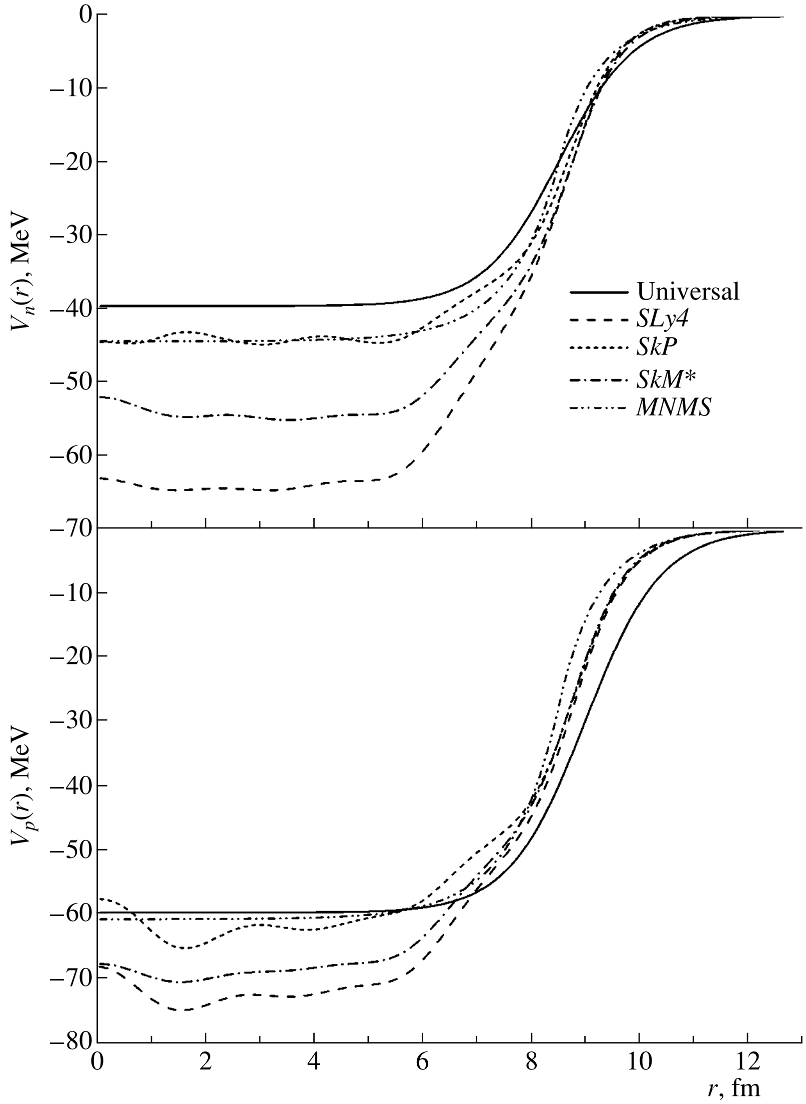

Fig. 1. Neutron and proton mean-field potentials for the $^{298}114$ nucleus within various models. The following notation is used here: the label "universal" corresponds to the Woods–Saxon potential whose parameters were borrowed from [27]; the symbols *SLy4*, *SkP*, and *SkM** denote the effective nucleon potentials determined in the Hartree–Fock approximation for the corresponding parametrizations of Skyrme forces; and MNMS stands for the mean-field potential from [14].

parameters are set to the values from the universal set and the potential borrowed from [14]. The potentials obtained in the Hartree–Fock approximation exhibit fluctuations in the interior of the nucleus.

We note that the depth and diffuseness parameter of the potentials obtained for the ^{208}Pb nucleus in the relativistic mean-field approximation [35] are close to the corresponding values calculated in the Hartree–Fock approximation with *SkM** or *SLy4* Skyrme forces. However, the potential well calculated in the relativistic mean-field approximation is wider than the potential wells obtained in the Hartree–Fock approximation with Skyrme forces.

The spin–orbit components of the potentials obtained within different models and approaches involving different versions of effective nucleon–nucleon forces also differ significantly from one another.

Potentials used in different models obviously lead to different values of shell corrections and different values of magic numbers in the region of superheavy nuclei (see Introduction). Shell corrections are small in relation to the total binding energy of the nucleus being considered, the latter being determined by the corresponding liquid-drop energy. Nevertheless, shell corrections affect considerably the accuracy in describing nuclear masses [13–15, 29, 36]. For example, currently used mass formulas [13–15, 25, 29, 37–39] yield a considerably smaller root-mean-square error than the value of shell corrections.

Ground-state shell corrections $E_{\text{shell}}^{\text{g.s}}$ make a decisive contribution to the fission barrier $B_f = B_{\text{macro}} + E_{\text{shell}}^{\text{bar}} - E_{\text{shell}}^{\text{g.s}} \approx B_{\text{macro}} - E_{\text{shell}}^{\text{g.s}} \approx -E_{\text{shell}}^{\text{g.s}}$ for superheavy nuclei lying along the beta-stability line since the shell correction at the barrier usually satisfies the condition $E_{\text{shell}}^{\text{bar}} \ll E_{\text{shell}}^{\text{g.s}}$, so that the macroscopic fission barrier B_{macro} is smaller than $E_{\text{shell}}^{\text{g.s}}$ [30]. Therefore, the fission barriers calculated for superheavy nuclei by using different potentials differ markedly [27, 40, 41]. We note that the experimental uncertainties in the fission barriers for superheavy nuclei are about 1 to 2 MeV [42].

Below, we will investigate the dependence of shell corrections on the variations in the parameters (depth, radius, and diffuseness) of the central and spin–orbit components of the Woods–Saxon potential. For the input parameters of the mean-field potential, we will employ values from the universal set introduced in [27, 43].

3. VARIATIONS IN THE MEAN FIELD AND SHELL CORRECTIONS

The central ($V(r)$) and the spin–orbit ($V_{\text{so}}(r)$) term in the spherical Woods–Saxon potential are parametrized in the form

$$V(r) = -\frac{V_0[1 \pm \kappa(N-Z)/(N+Z)]}{1+\exp[(r - r_0A^{1/3})/a]}, \qquad (2)$$

$$V_{\text{so}}(r) = \lambda \left(\frac{\hbar}{2mc}\right)^2 \qquad (3)$$

$$\times \frac{V_0 \exp[(r - r_{\text{so}}A^{1/3})/a_{\text{so}}]}{2ra_{\text{so}}\{1+\exp[(r - r_{\text{so}}A^{1/3})/a_{\text{so}}]\}^2}(\mathbf{l}\cdot\mathbf{s}).$$

In the numerator on the right-hand side of (2), a plus (minus) sign is taken for protons (neutrons); in (3), $\mathbf{s}$ and $\mathbf{l}$ stand for, respectively, the spin and the orbital nucleon operators, while $A = N + Z$, Z, and N are the numbers of, respectively, nucleons, protons, and neutrons in the nucleus being considered.

The universal set of V_0, r_0, a, λ, r_{so}, a_{so}, and κ values from [27, 43] describes well the single-particle levels in $A \geq 40$ even nuclei. Also, this set is successfully used to determine various properties of superheavy nuclei [27]. For this reason, we take this set as an input one in studying the effect of variations in the mean field on shell corrections. The single-particle energy levels, which are necessary for finding shell corrections, were calculated with the aid of the WSBETA code [43], which was modified to extend it to the case of a large number of nucleons in nuclei [16]. Shell corrections for protons and neutrons were calculated for $76 \leq Z \leq 140$ even–even nuclei in the vicinity of the stability line described by Green's formula [14, 16, 44]. We disregarded pairing forces and assumed a spherical shape of nuclei.

Figure 2 illustrates changes in shell corrections for protons and neutrons in response to variations in the parameter V_0. Specifically, these shell corrections were calculated for the value of V_0 from the universal set and for the values of $V_0 \pm 5$ MeV. The shell corrections for protons change insignificantly in the case of such variations in the potential depth, the position of deep minima at $Z = 82$ and 114, which correspond to magic numbers, remaining unchanged. The shell correction for protons exhibits shallow local minima at $Z = 120$ and 138. In [12], where use was made of a different set of mean-field parameters, the number $Z = 120$ of protons was also indicated as a possible magic number. Shell corrections for neutrons change only slightly in the vicinity of the magic number $N = 126$, but they exhibit sizable variations around the magic numbers $N = 184$ and 228. A variation in the depth of the neutron potential leads to an additional local minimum at $N = 164$. Further, a minimum at $N = 178$ appears as the potential depth increases. The behavior of the shell corrections for neutrons changes more strongly around $N = 228$ than in the vicinity of the magic number $N = 184$. Therefore, a variation in the depth of the central part of the mean-field potential does not affect shell corrections or magic numbers of protons in the region of superheavy nuclei; in the case of neutrons, changes in the shell corrections and magic numbers are small for superheavy nuclei. Insignificant changes in shell corrections in response to an increase in V_0 are due to insignificant variations in the potential in the vicinity of the Fermi level. It should be emphasized that, because of the sensitivity of the shell corrections to the level density in the vicinity of the Fermi energy, they must be sensitive to variations in the potential in this region.

Variations in the parameter κ give rise to variations in the depth V_0 of the central part of the mean-field potential. Since the V_0 dependence of the shell corrections was demonstrated in Fig. 2, we do not present a figure that would illustrate the effect of the choice of value for the parameter κ on the shell corrections.

Figures 3 and 4 display the shell corrections for protons and neutrons at various values of the radius r_0 and the diffuseness parameter a. The parameter r_0 determines the radius $R = r_0A^{1/3}$ of the central part of the potential for a nucleus that contains A nucleons, while the parameter a controls the diffuseness-layer thickness and the slope of the potential in the vicinity of the nuclear boundary. The shell corrections in question were calculated for the r_0 and a values recommended in the universal parametrization and for the values of $r_0 \pm 0.05$ fm and $a \pm 0.1$ fm. The shell corrections in the regions of actinides and superheavy

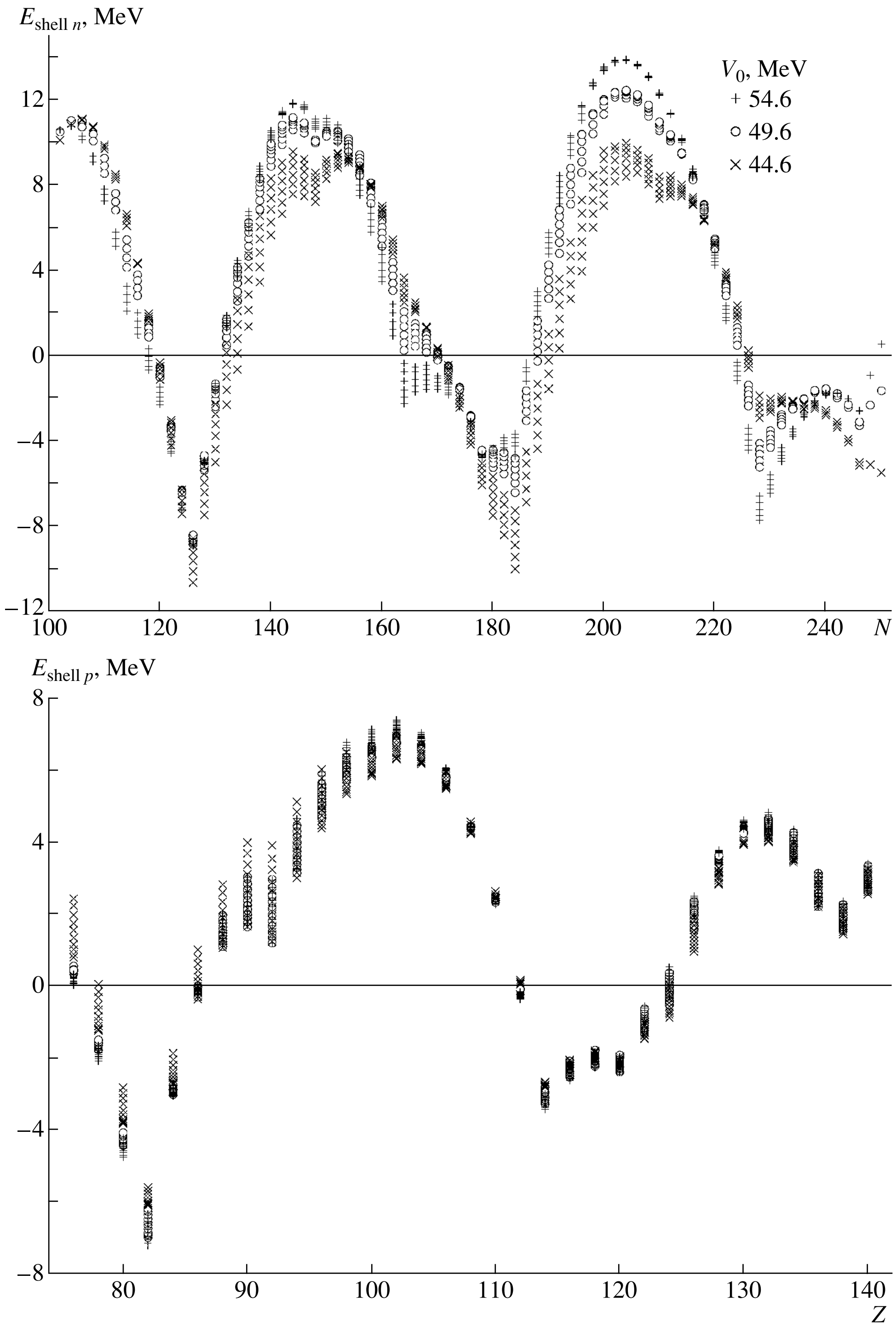

Fig. 2. Shell corrections for neutrons and protons in heavy and superheavy spherical nuclei versus the number of (upper panel) neutrons and (lower panel) protons for various values of the depth V_0 of the central part of the nucleon mean-field potential.

nuclei change markedly in response to variations in the parameters r_0 and a (see Figs. 3, 4). The position of deep minima also change significantly as the parameters r_0 and a become greater than the recommended values. In addition, there arise new local minima and, hence, the proton magic numbers of $Z = 120$ and 138 and the neutron magic numbers of $N = 164$ and 178. Thus, variations in the parameters r_0 and a of the central part of the mean-field potential affect substantially the magic numbers of protons and neutrons in the region of superheavy nuclei. However, these variations exert no effect on the magic numbers

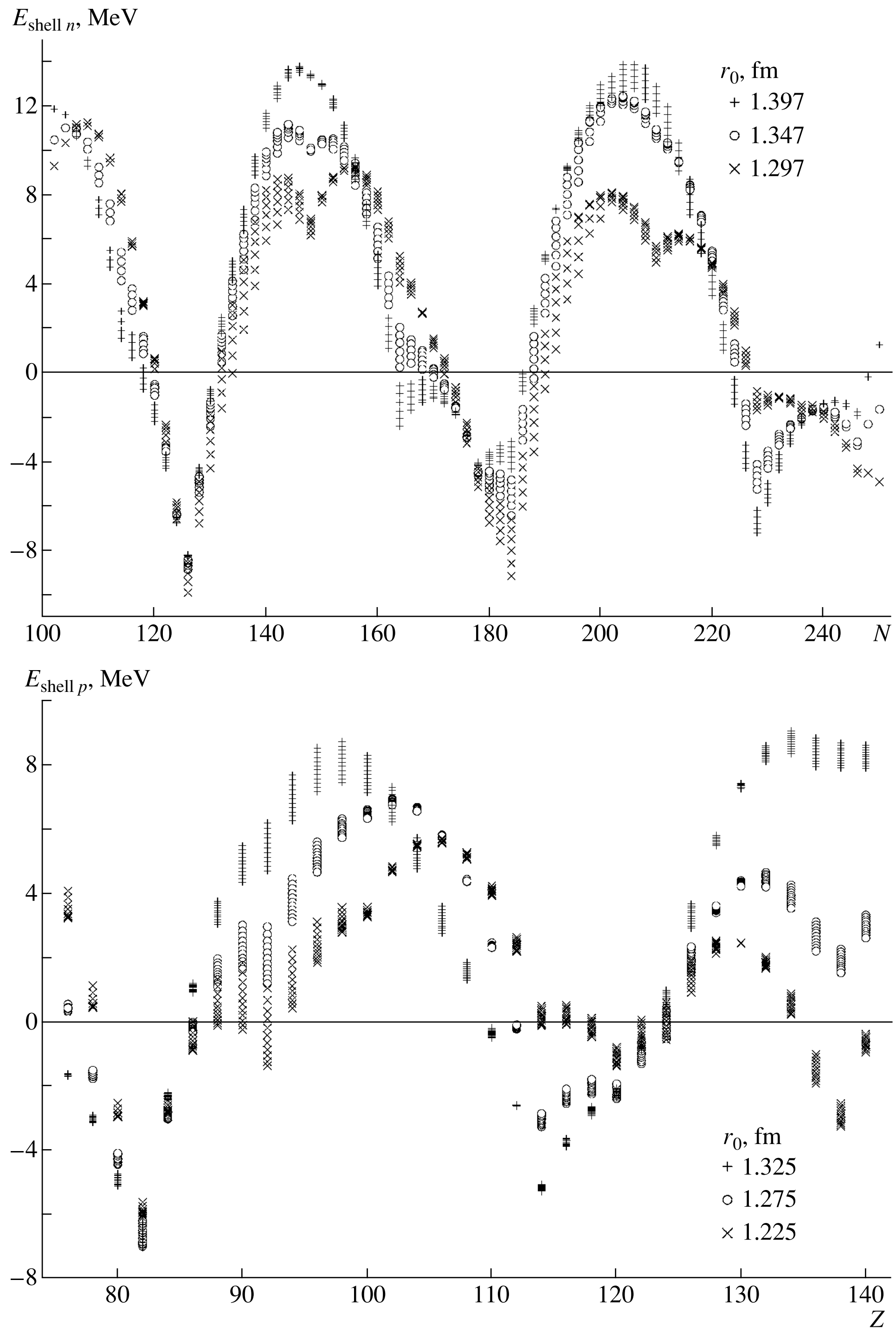

Fig. 3. Shell corrections for neutrons and protons in heavy and superheavy spherical nuclei versus the number of (upper panel) neutrons and (lower panel) protons for various values of the radius r_0 of the central part of the nucleon mean-field potential.

corresponding to the doubly magic nucleus of lead, $Z = 82$ and $N = 126$ (see Figs. 3 and 4).

In Figs. 5–7, we present the shell corrections for protons and neutrons at various values of the parameters λ, r_{so}, and a_{so} of spin–orbit-interaction. The shell corrections for neutrons and especially for protons change in response to variations in the strength λ and the radius r_{so} of the spin–orbit component of the mean field. The positions of the deep local minima in the shell correction for protons also change markedly in this case. Variations in λ and r_{so} have virtually no effect on the positions of the deep local minima in the

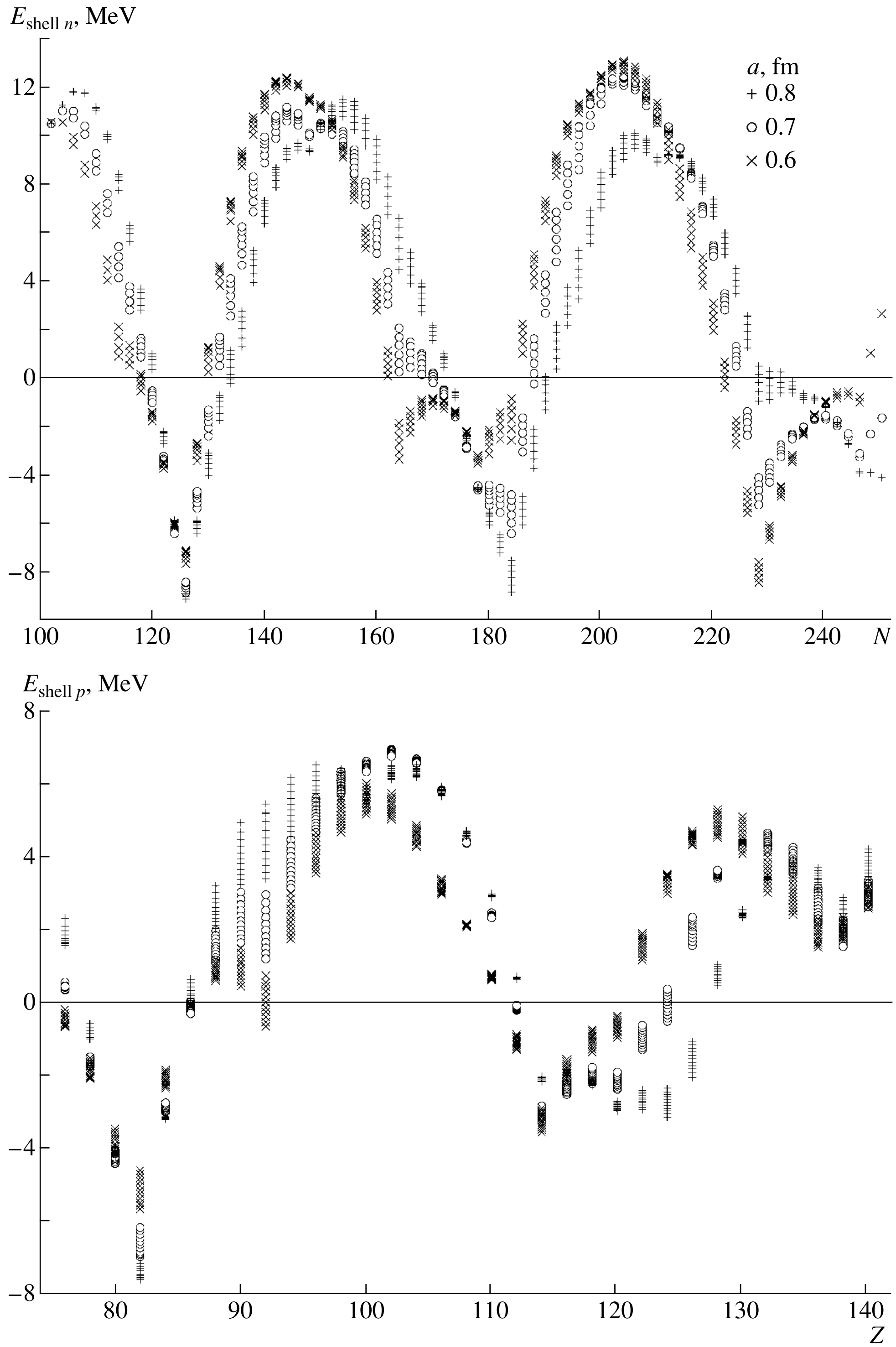

Fig. 4. Shell corrections for neutrons and protons in heavy and superheavy spherical nuclei versus the number of (upper panel) neutrons and (lower panel) protons for various values of the diffuseness parameter a of the central part of the nucleon mean-field potential.

shell corrections for neutrons and, hence, on magic numbers. Variations in the diffuseness parameter a_{so} of spin–orbit interaction affect only slightly the shell corrections both for protons and for neutrons.

Protons also interact with one another via the electric field. In the case of a spherical nucleus, the Coulomb potential of protons can be approximated by the electric potential of a uniformly charged sphere of

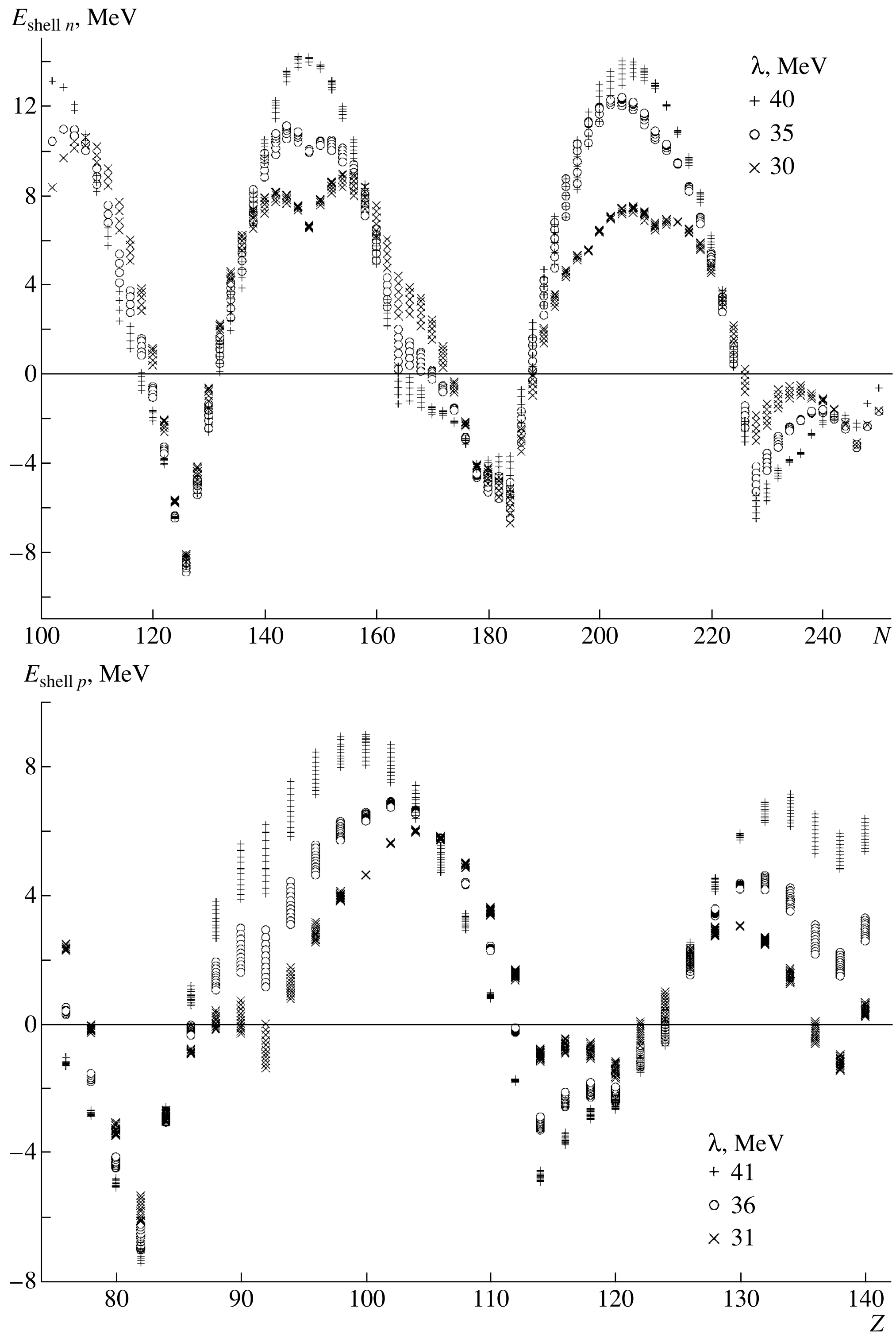

Fig. 5. Shell corrections for neutrons and protons in heavy and superheavy spherical nuclei versus the number of (upper panel) neutrons and (lower panel) protons for various values of the amplitude λ of the spin–orbit term in the nucleon mean-field potential.

radius R_c (for details, see [31, 43]). However, variations in R_c change only slightly the shell corrections for protons and do not shift the positions of minima in the shell corrections. Therefore, variations in the radius R_c do not affect the magic numbers in the region of heavy and superheavy nuclei.

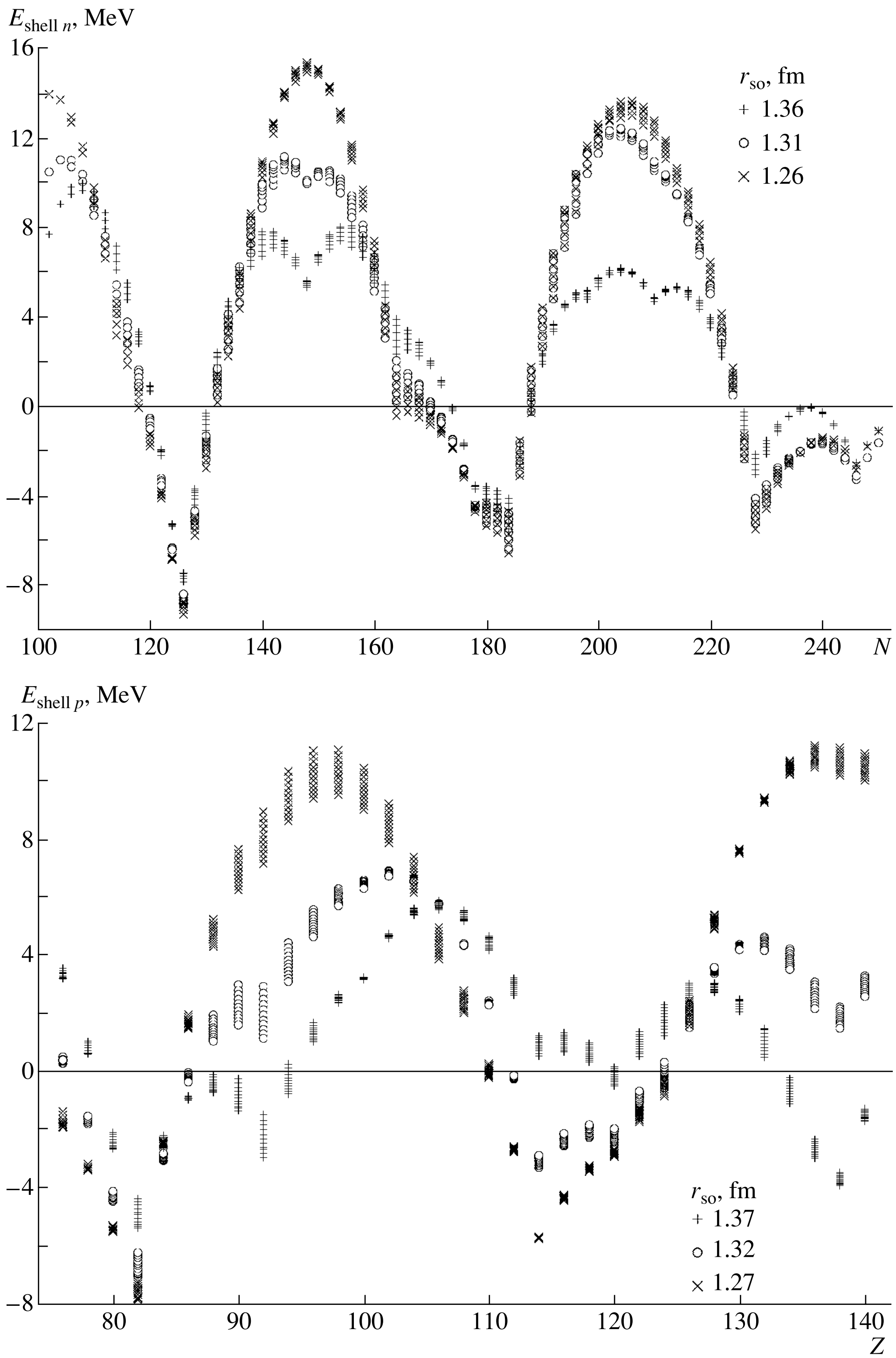

Fig. 6. Shell corrections for neutrons and protons in heavy and superheavy spherical nuclei versus the number of (upper panel) neutrons and (lower panel) protons for various values of the radius r_{so} of the spin–orbit term of the nucleon mean-field potential.

4. BINDING ENERGIES OF SUPERHEAVY NUCLEI WITHIN VARIOUS MODELS

As follows from the calculations presented in the preceding section, the magic numbers of protons and neutrons in spherical nuclei depend greatly on the parameters of the mean-field potential. Variations in these parameters lead to variations in the shell corrections; changes in the position of deep local minima;

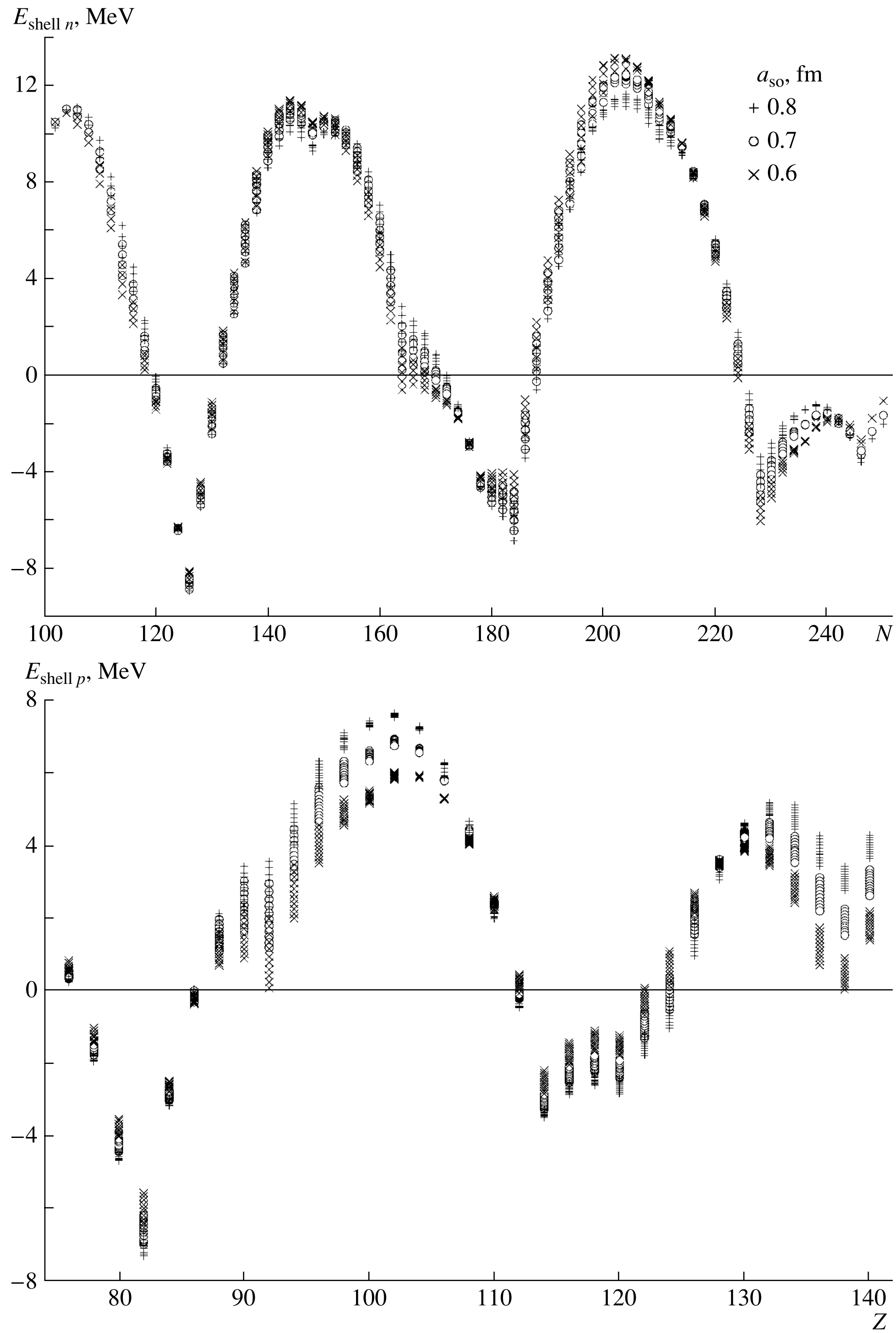

Fig. 7. Shell corrections for neutrons and protons in heavy and superheavy spherical nuclei versus the number of (upper panel) neutrons and (lower panel) protons for various values of the diffuseness parameter a_{so} of the spin–orbit term of the nucleon mean-field potential.

and the emergence of new minima and, hence, (new) magic numbers. Magic numbers that appear owing to variations in the parameters of the mean-field potential frequently coincide with their counterparts obtained within different models and approximations.

The accuracy of the shell-correction method and, Binding and excitation energies (in MeV units) of nuclei produced in the reaction ^{58}Fe + ^{208}Pb = ^{266}Hs = ^{265}He + n at the collision energy of 216 MeV (B and b are the absolute values of the binding energies of the ^{266}Hs and ^{265}Hs nuclei, respectively; δ is the energy of neutron separation from the ^{266}Hs nucleus; Q is the threshold energy for the formation of the compound nucleus ^{266}Hs; E^* is the excitation energy of the compound nucleus ^{266}Hs; and e is the maximum possible excitation energy of the compound nucleus ^{265}Hs)

B	b	δ	Q	E^*	e	References
1941.286	1933.235	8.051	205.092	10.908	2.857	[47]
1941.52	1933.37	8.15	204.858	11.14	2.99	[14]
1940.13	1931.90	8.23	206.248	9.75	1.52	[28]
1940.702	1932.130	8.571	205.676	10.324	1.752	[29]
1941.602	1933.630	7.971	204.776	11.224	3.252	[48]
1941.492	1933.650	7.841	204.886	11.114	3.272	[27]
1925.983	1917.932	8.051	220.395	−4.395	−12.446	[37][a]
1934.486	1926.435	8.051	211.895	4.108	−3.943	[37][b]
1930.822	1922.771	8.051	215.556	0.444	−7.607	[37][c]
1944.46	1936.41	8.051	201.918	14.082	6.031	[49]

[a] Results of the calculation with SkM^* Skyrme forces.
[b] Results of the calculation with SkP Skyrme forces.
[c] Results of the calculation with $SLy4$ Skyrme forces.

hence, of resulting magic numbers, nuclear masses, fission barriers, ground-state deformations, and other features of nuclei depends greatly on the choice of mean-field potentials and on their matching with nucleon-density distributions [12, 45]. (We do not discuss here the dependence of the accuracy in determining shell corrections on the parameter of averaging over levels, the proximity of the Fermi level to the continuous spectrum, and some other factors. These issues were considered in detail elsewhere [12, 46].) It follows that calculations in the Hartree–Fock or the relativistic mean-field approximation are the most accurate from the point of view of the self-consistency of the nucleon mean field. Nevertheless, the masses of nuclei are described more accurately in so-called macroscopic–microscopic approximations [27, 36], where the binding energy is represented as the sum of the liquid-drop energy and the shell correction.

Let us now consider the synthesis reaction ^{58}Fe + ^{208}Pb = ^{265}He + n, which was investigated experimentally in [11]. In the c.m. frame, the minimum collision energy at which this reaction occurs is about 216 MeV [10, 11, 30]. The table lists the binding energies of the ^{266}Hs and ^{265}Hs nuclei, the energy of neutron separation from the ^{266}Hs nucleus, and the excitation energies of the ^{266}Hs and ^{265}Hs nuclei that are produced at the collision energy of 216 MeV. The binding energies of the ^{266}Hs nucleus were taken from experimental data reported in [47] and results of calculations performed in various theoretical approximations [14, 27–29, 37, 48, 49]. In calculating the excitation energies of the ^{266}Hs and ^{265}Hs nuclei, we relied on the experimental binding energies of the ^{58}Fe and ^{208}Pb nuclei [47]. Only the masses of even–even nuclei were calculated in [37, 49]; therefore, the experimental energy of neutron separation from the ^{266}Hs nucleus [47] was used in the table to estimate the mass of the ^{265}Hs nucleus.

From the data in the table, one can see that the values obtained for the binding energy of the ^{266}Hs nucleus on the basis of macroscopic–microscopic models [14, 27–29] are close to its experimental value from [47]. The binding-energy values calculated in the Hartree–Fock–Bogolyubov approximation with *SkM**, *SkP*, and *SLy4* Skyrme forces [37] fall substantially short of this experimental value [47]. As a result, the threshold energy for the formation of the compound nucleus ^{266}Hs proves to be less than the energy at which the synthesis of ^{265}Hs in the reaction ^{58}Fe + ^{208}Pb = ^{265}He + n was successfully implemented in an experiment. A small value obtained for the binding energy of the ^{266}Hs nucleus in the calculations with *SkM**, *SkP*, and *SLy4* Skyrme forces leads to negative values of E^* and e in the table. We note that the *MSk7* parametrization of Skyrme forces, which was obtained from a fit to nuclear masses, is free from this drawback, the binding energies calculated with these forces in the Hartree–Fock–Bardeen–Cooper–Schriefer approximation satisfying experimental conditions.

Taking into account the energy dependence of the ratio of the width with respect to neutron emission from the compound nucleus ^{266}Hs to its fission width [30], we can conclude that, for the reaction $^{58}\mathrm{Fe} + {}^{208}\mathrm{Pb} = {}^{266}\mathrm{Hs} = {}^{265}\mathrm{He} + n$ at a collision energy of about 216 MeV to proceed successfully, the excitation energy of the ^{266}Hs nucleus must not be smaller than 9 MeV. If use is made of *SkM**, *SkP*, and *SLy4* Skyrme forces, the binding energy of the ^{266}Hs nucleus is so small that experimental conditions cannot be met at a collision energy of about 216 MeV. This indicates that the liquid-drop part of the energy is described inadequately in the Hartree–Fock approximation with *SkM**, *SkP*, and *SLy4* Skyrme forces. Shell-correction variations caused by changes in the mean field are insufficient for obtaining so large a variation in the binding energy of the ^{266}Hs nucleus in the case of the above forces (this would lead to a value of 9 to 10 MeV for the excitation energy of the ^{266}Hs nucleus at a collision energy of about 216 MeV). We note that the shell corrections $E_{\mathrm{shell}\ p} + E_{\mathrm{shell}\ n}$ in the ground states of the ^{266}Hs and ^{265}Hs nuclei are, respectively, −6 and −5.2 MeV [27].

Since microscopic calculations employing some parametrizations of Skyrme forces lead to an unsatisfactory description of binding energies for superheavy nuclei, there is a doubt as to whether the predictions of the respective models for magic numbers are correct.

Calculations in the Hartree–Fock approximation with Skyrme forces satisfactorily describe binding energies, nucleon-density distributions, and some other features of medium-mass and heavy nuclei in the ground and excited states [18]. The values obtained for the radii of the nucleon mean-field potentials in self-consistent calculations involving various parametrizations of Skyrme forces are quite realistic for medium-mass, heavy, and possibly superheavy nuclei. As was indicated previously, the proton and especially neutron mean fields in the form of the Woods–Saxon potential whose parameters are set to the values from the universal set have the largest well radii among the potentials presented in Fig. 1. On the contrary, the mean field determined in the approximation where the Yukawa potential is folded with a stepwise density distribution has the smallest radius. Therefore, the form of these empirical potentials, which are used to calculate shell corrections in the macroscopic–microscopic models, agrees poorly with the nucleon-density distribution obtained within self-consistent microscopic approaches. Thus, the inconsistency of the form of potential with the density distribution reduces the predictive power of the shell-correction method and impairs the accuracy in describing nuclear masses and fission barriers. If use is made of macroscopic–microscopic models, the values of shell corrections and regularities in their behavior in response to variations in both the number of nucleons and the deformations of the nuclear shape can therefore appear to be unrealistic. We note that, in the model proposed in [27], a very high accuracy in describing the masses of heavy and superheavy nuclei is achieved by fitting the parameters of the liquid-drop energy and by restricting the interval in which nuclear masses are described.

5. CONCLUSIONS

The main conclusions of the present study can be briefly formulated as follows:

In the region around $Z = 82$ and $N = 126$, variations in the parameters of the mean-field potential affect only slightly the values of shell corrections. The position of the deep local minimum in the shell corrections at $Z = 82$ and $N = 126$ is stable against variations in the parameters of the mean-field potential.

In the region of superheavy nuclei, variations in the parameters of the mean-field potential change substantially the shell corrections and the positions of their local minima, this leading to a change in magic numbers. The effect of variations in the mean field on the shell corrections and magic numbers becomes stronger with increasing Z and N.

Variations in shell corrections in response to employing different nucleon mean-field potentials lead to different values of the magic numbers and to the dependence of nuclear masses and fission barriers on the number of nucleons in superheavy nuclei.

Shell corrections are the most sensitive to variations in the parameters r_0, a, λ, and r_{so} [see formulas (2) and (3)], since these variations have the strongest effect on the mean-field potential in the vicinity of the Fermi level.

For heavy and superheavy nuclei, shell corrections for protons are usually more sensitive to variations in the parameters of the mean-field potential than shell corrections for neutrons.

In the region of superheavy nuclei, magic numbers obtained within various self-consistent microscopic models can be reproduced by using the Woods—Saxon potential whose parameters are chosen appropriately.

In the region of superheavy nuclei, the calculations in the Hartree—Fock approximation with *SkM**, *SkP*, and *SLy4* Skyrme forces lead to underestimated values for the liquid-drop component of the binding energy.

A high sensitivity of the structure and binding energies of superheavy nuclei to the parametrization of nuclear forces can be used to refine parametrizations of effective nucleon—nucleon forces.

In order to obtain a more reliable description of binding energies and shell corrections within macroscopic—microscopic models, it is necessary to match the nucleon-density distribution and the potential used.

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