Nuclear reactions in hot stellar matter and nuclear surface deformations

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Cross sections for capture reactions of charged particles in hot stellar matter turn out be increased by the quadrupole surface oscillations when the corresponding phonon energies are of the order of the star temperature. This increase is studied in a model that combines barrier distribution induced by surface oscillations and tunneling. The capture of charged particles by nuclei with well-deformed ground states is enhanced in stellar matter. It is found that the influence of quadrupole surface deformation on the nuclear reactions in stars grows when mass and proton numbers in colliding nuclei increase.

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I. INTRODUCTION

Various nuclear reactions take place in the stars [1–16]. The cross sections of nuclear reactions determine diverse properties of the stars [1–3,5,14] and the nucleosynthesis of elements in stellar matter [1–13]. Nuclear reactions occur at very high temperatures during the burning of massive stars, in supernova explosions, or in the crust of neutron stars [1,3–5]. A typical temperature of the silicon burning of massive stars is close to 3.5×10^9K ≈ 0.3 MeV [5]. The star matter at temperatures $\sim$0.3 MeV mainly consists of the most tightly bound iron-group nuclei, α particles, nucleons, electrons, and γ quanta [1,2,4,5,10]. Subsequent stages of the evolution of massive stars may take place at higher temperatures. The composition, density, and temperature of stellar matter depend on the mass, as well as on the type and age of the star [1–5].

Nuclei in the star participate in various reactions induced by γ quanta, electrons, free nucleons, and other nuclei. Therefore, nuclei in star matter exist in both the ground and excited states due to photoexcitation, i.e., inelastic collisions between the nuclei and electrons, free nucleons, and other nuclei. At star temperatures $\gtrsim$0.2 MeV, the population probability of low-energy (0.2–1 MeV) excited states in soft nuclei is rather noticeable. Such states are exemplified by the lowest 2^+ surface oscillation states in ^{52}Fe and ^{80}Sr with excitation energies, respectively, of $\varepsilon_{\rm vib} = 0.849$ and 0.386 MeV [17]. The total vibrational amplitudes of these states are large, namely, $\beta_{\rm vib} = 0.308$ and 0.404 [17]. Therefore, in evaluating the reaction cross sections between a charged particle and heavy soft nucleus in star matter, we should take into account contributions from both the ground and well-deformed excited states of the nucleus.

The astrophysical importance of charged-particle capture on target nuclei with $N \cong Z$ is manifold. These capture reactions are important for nucleosynthesis processes in stellar matter and various burning phases of massive stars [2,4,5,7,8].

Cross sections of nuclear reactions may be evaluated using the nucleus-nucleus or nucleon-nucleus potentials [2,18,19]. The nucleus-nucleus potential depends on the shape of nuclei participating in the reaction [18–24]. In the case of the interaction between spherical and prolate nuclei, the barrier for the tip orientation of deformed nucleus is smaller than one for the side orientation. For example, these barriers for the system ^{48}Ca+^{238}U are, respectively, close to 184 and 202 MeV for corresponding orientation of ^{238}U [20], while the value of the barrier evaluated for the spherical shape of ^{238}U is 197 MeV [25]. Similar reduction of the barrier induced by deformation strongly enhances the subbarrier fusion cross sections for light- and medium-weight collision systems [19,21–24]. Note that charged-particle capture on heavy nuclei in stellar matter takes place at subbarrier energies at temperatures $\lesssim$0.5 MeV. Therefore, the enhancement of capture rates caused by deformation of heavy nuclei should be important for various reactions in stars.

The 2^+ shape oscillations in soft nuclei may also affect nuclear reaction rates in the hot stars, because the energies of such states are small, and low-energy nuclear states can be appreciably populated in stellar matter at temperatures $\gtrsim$0.2 MeV. The amplitude of surface deformation in soft nuclei is large. Consequently, the reduction of fusion barrier induced by this deformation can be significant. Cross sections of the fusion (capture) reaction in stars can be enhanced essentially by the barrier reduction caused by 2^+ shape oscillations. Therefore, quadrupole shape oscillations in soft nuclei should be taken into account to accurately evaluate reaction cross sections in hot star matter.

The influence of the star environment and of specific properties of nuclei on nuclear reactions in stars has been taken into account in some calculations in nuclear astrophysics [6,12,13]. However, the p- and α-capture reactions on deformed nuclei are usually treated by means of an effective spherical potential of equal volume [7,8,13]. We consider enhancement of cross sections for various nuclear reactions in hot stellar matter induced by the low-energy 2^+ oscillations of the nuclear shape, because the barrier between nuclei is reduced as a result of the shape deformation induced by the 2^+ oscillations. As the result, the probability of subbarrier tunneling is strongly increased at smaller values of the barrier height. Such a mechanism of nuclear reaction enhancement in stars has not yet been considered. Our consideration includes α and p capture, as well as heavy-ion fusion reactions for soft nuclei.

Reactions α+Ne, α+Mg, α+Si, and some other are very important for the O-Si burning phases of massive stars [1,2,4,5,8]. Heavy nuclei participating in these reactions are well deformed in the ground state. Many other proton-rich nuclei participating in various charged-particle capture reactions in stellar matter are also well deformed in the ground state. We include the ground-state deformation of the heavy nucleus into the interaction potentials between the charged particle and heavy nucleus. In the framework of such treatment, we discuss cross-section enhancement caused by the ground-state deformation of heavy nuclei for various reactions in stellar matter.

This article is organized as follows. Partition probabilities of the ground and 2^+ surface oscillation states of nuclei in stellar matter are discussed in Sec. II. Influence of low-energy quadrupole vibrations of the nuclear surface on both S factors and the velocity-averaged cross sections of α-nucleus capture reactions in stars is considered in Sec. III. The effect of 2^+ surface oscillations on proton capture and nucleus-nucleus fusion reactions in stellar matter is studied in Secs. IV and V, respectively. The capture of charged particles by nuclei with well-deformed ground states is discussed in Sec. VI. Conclusions are presented in Sec. VII.

II. PARTITION PROBABILITY OF NUCLEAR STATES IN STELLAR MATTER

The probability of finding a nucleus in a state with excitation energy ε_i and spin j_i in stellar matter at temperature T can be estimated within the statistical approach as

$$P(\varepsilon_i, j_i, kT) = \frac{(2j_i + 1)\exp(-\varepsilon_i/kT)}{\sum_{i=0}^{\infty}(2j_i + 1)\exp(-\varepsilon_i/kT)}. \quad (1)$$

Here k is the Boltzmann constant. We use $i = 0$ for the ground state of the nucleus with $\varepsilon_i = 0$, $i = 1$ for the lowest 2^+ surface oscillation state with $\varepsilon_1 = \varepsilon_{\rm vib}$ and $i \geqslant 2$ for other excited states.

The partition probabilities of both the ground and the lowest 2^+ surface oscillation states in ^{52}Fe and ^{80}Sr nuclei for different temperatures of stellar matter are presented in Fig. 1. The partition probabilities are evaluated by using the experimental

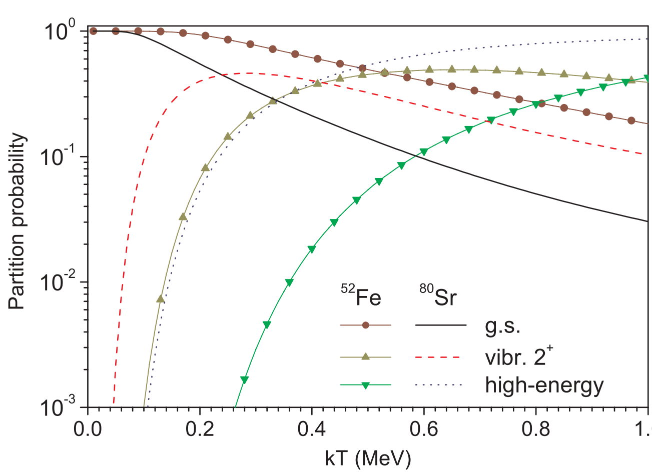

FIG. 1. (Color online) Occupation probability for the ground state and first 2^+ surface oscillation state and the net occupation probability of high-energy states $P(\varepsilon_i > \varepsilon_{\rm vib})$ in ^{52}Fe and ^{80}Sr at different temperatures of stellar matter kT.

data for the lowest 98 states in ^{52}Fe (up to 11.780 MeV) and 90 states in ^{80}Sr (up to 15.576 MeV) listed in compilation [26]. In addition to energies, values of spin for many levels are also given in this data compilation [26]. If the value of spin for level i is not listed, then we assign $j_i = 0$. On the other hand, when the spin is not well defined from experiments and several possible values of spin are listed for a level in Ref. [26], we choose the maximal value among the listed values. It should be mentioned that this choice for spin assignments is of marginal practical importance for specific population probabilities displayed in Fig. 1. Those depend primarily on the spins of the lowest states. The spins of 6 and 3 lowest states in ^{52}Fe and ^{80}Sr, respectively, are well established [26].

The total occupation probability of high-energy states with energies $\varepsilon_i > \varepsilon_{\rm vib}$ is equal to

$$P(\varepsilon_i > \varepsilon_{\rm vib}) = 1 - \sum_{i=0,1} P(\varepsilon_i, j_i, kT).$$

We see in Fig. 1 that the net occupation probability of high-energy states becomes dominant at very high temperatures of the stellar matter, $kT \gtrsim 0.5$ MeV. As seen further in Fig. 1, the partition of 2^+ surface oscillation states is negligible for small temperatures $kT \lesssim 0.1$ MeV. On the other hand, the 2^+ vibrational states are noticeably populated at temperatures $0.2 \lesssim kT \lesssim 0.7$ MeV. Moreover, the occupation for vibrational states is higher than for the ground states at temperatures $kT > 0.53$ MeV for ^{52}Fe and at $kT > 0.24$ MeV for ^{80}Sr. Therefore, nuclear reaction cross sections at moderate temperatures of stellar matter should be evaluated by taking into account properties of both the ground and 2^+ states. Nuclei ^{52}Fe and ^{80}Sr are spherical in the ground state but generally deformed in the first 2^+ surface vibrational states. Thus it is necessary to consider the nuclear reactions both for spherical and deformed shapes of nuclei ^{52}Fe and ^{80}Sr in hot star matter.

The cases of the nuclei ^{52}Fe and ^{80}Sr are interesting from the point of view of the role of deformation in reactions. On the one hand, energies of the 2^+ surface oscillations $\varepsilon_{\rm vib} \equiv \varepsilon_1$ are low, making the occupations for these states significant in the hot stellar medium. On the other hand, the stiffness of the nuclear surface with respect to deformations $C_{\rm vib}$ is also low for these nuclei, making the shape vibration amplitudes $\beta_{\rm vib} = [5\varepsilon_{\rm vib}/(2C_{\rm vib})]^{1/2}$ large. We consider 2^+ surface oscillations in nuclei in the framework of the harmonic oscillator model [27].

There is a variety of nuclei in stellar matter. Nuclei far from the β-stability line are soft as a rule. Nuclei with a number of nucleons, being far from the magic numbers, and lying along the β-stability line are often soft too. However, many rigid nuclei such as ^{4}He, ^{12}C, ^{16}O, ^{40,48}Ca, ^{208}Pb also appear in hot stellar matter. Energies of the 2^+ surface oscillation states are rather high in those nuclei, making the 2^+ states weakly populated in the stellar matter. Shape vibration amplitudes are small in rigid nuclei. As a result, the influence of the 2^+ surface vibrations for such rigid nuclei on nuclear reactions in stars may be neglected. Overall, it is necessary to develop a formalism for nuclear reactions in stellar matter that is applicable to both to soft and rigid nuclei.

III. α-CAPTURE REACTIONS IN STARS

The α particle may be considered the most rigid nucleus, because the energy of the first excited state is 20.21 MeV [26]. The population of such a high-energy state is negligible in stellar matter at temperatures $kT \lesssim 1$ MeV. On the other hand, many states with energies $\varepsilon_i \lesssim 1$ MeV in soft nuclei can be noticeably populated at $kT \lesssim 1$ MeV. Therefore, in evaluating α-capture reaction cross sections in stellar matter at $kT \sim 0.3$ MeV, we should take into account the contributions of various populated states in soft nuclei only. Correspondingly, the α-capture reaction cross section in star matter at temperature T can be estimated as

$$\sigma(E, kT) = \sum_{i=0}^{\infty} P(\varepsilon_i, j_i, kT)\sigma_i(E), \qquad (2)$$

where $\sigma_i(E)$ is the fusion cross section between the α particle and a nucleus in a state i with energy ε_i and spin j_i, and E is the collision energy.

We shall consider the α capture on nuclei ^{52}Fe and ^{80}Sr as an example. The reaction $\alpha + {}^{52}\mathrm{Fe} \Rightarrow {}^{56}\mathrm{Ni}$ may take place in the last stage of the silicon burning of the stellar matter, when iron nuclei are the most abundant nuclei [5,6]. Reaction $\alpha + {}^{80}\mathrm{Sr} \Rightarrow {}^{84}\mathrm{Zr}$ is important for nucleosynthesis. Note that α-capture reactions on nuclei with $N \cong Z$ are substantial for nucleosynthesis processes in stellar and explosive burning [5–8].

As we have seen in the previous section, the ground state and the first 2^+ surface oscillation state of a heavy nucleus are mainly populated in star matter at moderate temperatures. The nuclei ^{52}Fe and ^{80}Sr are spherical in the ground state and generally deformed in the first 2^+ state.

The α-capture reaction cross section in star matter at temperature T consists of three terms

$$\sigma(E, kT) = P(0, 0, kT)\sigma_0(E) + P(\varepsilon_{\mathrm{vib}}, 2, kT)\sigma_1(E) + \sum_{i=2}^{\infty} P(\varepsilon_i, j_i, kT)\sigma_i(E), \qquad (3)$$

where $P(0, 0, kT)$ and $P(\varepsilon_{\mathrm{vib}}, 2, kT)$ are the partition probabilities for the ground state and the lowest 2^+ surface oscillation state, respectively, while $\sigma_0(E)$ and $\sigma_1(E)$ are the cross sections of α capture on the heavy nucleus in the ground state and the lowest 2^+ vibrational state, correspondingly.

The fusion cross section of two particles with corresponding values of spins 0 and j is given by [18]

$$\sigma(E) = \frac{\pi\hbar^2}{2\mu E(2j+1)} \sum_{J\ell\ell'} (2J+1) t_{J\ell\ell'}(E), \qquad (4)$$

where μ is the reduced mass, ℓ and ℓ' are the orbital moment of ingoing and outgoing channels, J is the total angular momentum, and $t_{J\ell\ell'}(E)$ is the generalized transmission coefficient. If there is no spin-orbit forces in the potential, then the transmission coefficient becomes independent on J and $t_{J\ell\ell'}(E) = \delta_{\ell\ell'} t(E, \ell)$ [18]. In this case, Eq. (4) can be simplified as

$$\begin{aligned}\sigma(E) &= \frac{\pi\hbar^2}{2\mu E(2j+1)} \sum_{J\ell} (2J+1) t(E, \ell) \\ &= \frac{\pi\hbar^2}{2\mu E(2j+1)} \sum_{\ell} t(E, \ell) \sum_{J=|\ell-j|}^{\ell+j} (2J+1) \\ &= \frac{\pi\hbar^2}{2\mu E} \sum_{\ell} (2\ell+1) t(E, \ell), \end{aligned} \qquad (5)$$

where $t(E, \ell)$ is the transmission coefficient for particles penetration through the interaction potential barrier between them at collision energy E and orbital momentum ℓ.

Any indications on the spin-orbit interaction between the α particle and heavy nucleus are not known yet. The ground-state spin of the α particle is zero. Therefore, we apply Eq. (5) to the evaluation of the α-capture cross section on a heavy nucleus in the ground and excited states.

The ground-state shape of α particles ^{52}Fe and ^{80}Sr is spherical; therefore, the α-capture cross section can be evaluated in the usual manner [see Eq. (5) and Refs. [18,19,23,28]],

$$\sigma_0(E) = \frac{\pi\hbar^2}{2\mu E} \sum_{\ell} (2\ell+1) t_0(E, \ell). \qquad (6)$$

Here $t_0(E, \ell)$ is the transmission coefficient for penetration through the interaction potential barrier between the spherical nucleus in the ground state and the α particle at collision energy E.

The cross section of α capture on a heavy nucleus in the 2^+ vibrational state is also obtained by using Eq. (5). However, the surface of the nucleus in the 2^+ state oscillates around a spherical equilibrium shape. Therefore, we should take into account both the orientation and the surface oscillation effects when evaluating the transmission coefficient in this case.

During the α-nucleus fusion reaction, the α particle can arrive from any direction; therefore, we should average over the space angles. In the case of collision between spherical and axially deformed nuclei, averaging over the space angle is reduced to averaging over the angle θ, where θ is the angle between the symmetry axis of the axially symmetric deformed nucleus and the vector directed from the center of the deformed nucleus to the center of the α particle. Therefore, the fusion reaction cross section between the nucleus with axial quadrupole surface vibrations and the α particle at collision energy E equals

$$\sigma_1(E) = \frac{\pi\hbar^2}{2\mu E} \sum_{\ell} (2\ell+1) \int_0^{\pi/2} t_1(E, \ell, \theta) \sin(\theta) d\theta \qquad (7)$$

(see also Ref. [23]). Here $t_1(E, \ell, \theta)$ is the transmission coefficient, which shows the probability of penetration through the potential barrier for an α particle coming in at angle θ.

The nuclear surface in the 2^+ state in ^{52}Fe and ^{80}Sr oscillates about the spherical equilibrium shape. The distance between the deformed nuclear surface and the origin is

$$R(\theta) = R_0(1 + \beta Y_{20}(\theta)), \qquad (8)$$

where R_0 is the radius of the spherical nucleus, β is the deformation parameter, and $Y_{20}(\theta)$ is the spherical harmonic function. The distribution of the deformation parameter values in the 2^+ state is described by the square of the vibrational wave function $\varphi_{\rm vib}(\beta)$, i.e.,

$$D(\beta) = |\varphi_{\rm vib}(\beta)|^2 = \frac{\beta^2}{\sqrt{2\pi}\,\beta_0^3}\exp\left(-\frac{\beta^2}{2\beta_0^2}\right), \tag{9}$$

where $\beta_0 = \beta_{\rm vib}/\sqrt{5}$ is the zero-point amplitude. Here we use the harmonic oscillator model of the 2^+ surface vibrational state in nuclei [27]. Because the value of β can be different during the barrier penetration, we should average over all possible values of the deformation parameter. As a result,

$$t_1(E,\ell,\theta) = \int_{-\infty}^{\infty} D(\beta) t_1(E,\ell,\theta,\beta)\, d\beta, \tag{10}$$

where $t_1(E,\ell,\theta,\beta)$ is the transmission coefficient of a charged particle coming in at angle θ and evaluated at the value of surface deformation β. [Here we consider the evaluation of the cross section and other related quantities in the framework of the time-independent scattering theory [19]. Therefore, the time-independent oscillator wave function $\varphi_{\rm vib}(\beta)$ is used in Eqs. (9)–(10). The averaging over β in Eq. (10) is equivalent to time averaging over the period of the nuclear surface oscillations in the framework of time-dependent scattering theory [19].]

We estimate the transmission coefficients in Eqs. (6) and (10) using the semiclassical WKB approximation at collision energies below the barrier, that is,

$$t_0(E,\ell) = \left\{1 + \exp\left[\frac{2}{\hbar}\int_a^b dr\sqrt{2\mu\,(v_0(r,\ell,E)-E)}\right]\right\}^{-1}, \tag{11}$$

$$t_1(E,\ell,\theta,\beta) = \left\{1 + \exp\left[\frac{2}{\hbar}\int_{a(\theta)}^{b(\theta)} dr\sqrt{2\mu\,(v_1(r,\ell,\theta,E,\beta)-E)}\right]\right\}^{-1}. \tag{12}$$

Here $v_0(r,\ell,E)$ and $v_1(r,\ell,\theta,E,\beta)$ are the interaction potentials between the α particle and nucleus in the spherical ground state and deformed 2^+ state, respectively; r is the distance between the mass centers of colliding particles; and a, $a(\theta)$ and b, $b(\theta)$ are the inner and outer turning points determined from the corresponding equations $v_0(r,\ell,E)|_{r=a,b} = E$ and $v_1(r,\ell,\theta,E,\beta)|_{r=a(\theta),b(\theta)} = E$. The transmission coefficients $t_0(E,\ell)$ and $t_1(E,\ell,\theta,\beta)$ are approximated by an expression for a parabolic barrier at collision energies higher than the barrier energy.

We propose that parameters of α-nucleus interactions are the same for the ground and vibrational states; therefore, $v_0(r,\ell,E) = v_1(r,\ell,\theta,E,\beta=0)$. Because of this, we omit indexes 0 and 1 for potentials below.

The interaction potential between the deformed nucleus and charged particle $v(r,\ell,\theta,\beta)$ consists of Coulomb $v_C(r,\theta,\beta)$, nuclear $v_N(r,\theta,\beta)$, and centrifugal $v_\ell(r)$ parts,

$$v(r,\ell,\theta,E,\beta) = v_C(r,\theta,\beta) + v_N(r,\theta,E,\beta) + v_\ell(r). \tag{13}$$

The Coulomb part of the interaction potential between the α particle and deformed nucleus takes into account the effect of deformation to the first order,

$$v_C(r,\theta,\beta) = \frac{zZe^2}{r}\left[1 + \frac{3R_0^2}{5r^2}\beta Y_{20}(\theta)\right], \tag{14}$$

if $r \geqslant r_m(\theta,\beta)$, and

$$v_C(r,\theta,\beta) \approx \frac{zZe^2}{r_m(\theta,\beta)}\left[\frac{3}{2} - \frac{r^2}{2r_m(\theta,\beta)^2} + \frac{3R_0^2}{5r_m(\theta,\beta)^2}\beta Y_{20}(\theta)\left(2 - \frac{r^3}{r_m(\theta,\beta)^3}\right)\right], \tag{15}$$

if $r \lesssim r_m(\theta,\beta)$. Here $z = 2$ is the charge of the α particle, Z is the number of protons in the nucleus, and $r_m(\theta,\beta)$ is the effective radius of the nuclear part of the α-nucleus potential. The inner turning point $a(\theta)$ is close to $r_m(\theta,\beta)$; therefore, the representation of the Coulomb field in the form (15) at distances $r \lesssim r_m(\theta)$ ensures continuity of the Coulomb field and its derivative at $r = r_m(\theta,\beta)$ [23].

The nuclear part of the α-nucleus interaction is taken in the Woods-Saxon shape as

$$v_N(r,\theta,E,\beta) = \frac{V(A,Z,E)}{1+\exp[(r-r_m(\theta,\beta))/d]}, \tag{16}$$

with parameters [23]

$$V(A,Z,E) = -(30.275 - 0.45838Z/A^{1/3} + 58.270I - 0.24244E), \tag{17}$$

$$r_m(\theta,\beta) = 1.5268 + R(\theta) = 1.5268 + R_0(1+\beta Y_{20}(\theta)), \tag{18}$$

$$R_0 = R_p\left(1 + 3.0909/R_p^2\right) + 0.12430t, \tag{19}$$

$$R_p = 1.24A^{1/3}(1 + 1.646/A - 0.191I), \tag{20}$$

$$t = I - 0.4A/(A+200), \tag{21}$$

$$d = 0.49290. \tag{22}$$

Here A is the number of nucleons in a nucleus, and $I = (A-2Z)/A$.

The rotational part of the interaction is

$$v_\ell(r) = \hbar^2\ell(\ell+1)/(2\mu r^2). \tag{23}$$

Data for both the α-decay half-lives and the fusion cross sections around the barrier for reactions $\alpha + {}^{40}$Ca, $\alpha + {}^{59}$Co, and $\alpha + {}^{208}$Pb are well described by using this parametrization of the α-nucleus potential [23].

The values of the potential barrier $v_{\rm bar}(\theta)$ between spherical and deformed ($\beta \neq 0$) nuclei at various angles θ and $\ell = 0$ obey the inequalities

$$v_{\rm bar}(\theta=0)|_{\beta>0} < v_{\rm bar}^{\rm sph} < v_{\rm bar}(\theta=\pi/2)|_{\beta>0}, \tag{24}$$

$$v_{\rm bar}(\theta=0)|_{\beta<0} > v_{\rm bar}^{\rm sph} > v_{\rm bar}(\theta=\pi/2)|_{\beta<0}, \tag{25}$$

where $v_{\rm bar}^{\rm sph}$ is the barrier of the interaction potential if both nuclei are spherical. The barrier of the interaction potential $v(r,\ell,\theta,E,\beta)$ is reduced by quadrupole surface distortion in one or both interacting nuclei. As an example, the values of the barrier for reactions $\alpha + {}^{52}\mathrm{Fe}$ and $\alpha + {}^{80}\mathrm{Sr}$ for the spherical ground state at 15 MeV collision energy are $v_{\mathrm{bar}}^{\mathrm{sph}} = 7.98$ and 11.13 MeV, respectively. If nuclei $^{52}\mathrm{Fe}$ and $^{80}\mathrm{Sr}$ are deformed (let $\beta = \beta_{\mathrm{vib}}$), these barrier values are distributed over the ranges $v_{\mathrm{bar}}(\theta = 0, \beta = 0.308) = 7.35$ to $v_{\mathrm{bar}}(\theta = \pi/2, \beta = 0.308) = 8.30$ MeV and $v_{\mathrm{bar}}(\theta = 0, \beta = 0.404) = 9.97$ to $v_{\mathrm{bar}}(\theta = \pi/2, \beta = 0.404) = 11.70$ MeV, correspondingly. The reduction of the barrier height by ∼0.5–1 MeV induced by deformation $\beta = \beta_{\mathrm{vib}}$ increases the transmission coefficient at subbarrier energies for θ close to 0. As a result, the fusion cross section is strongly enhanced for subbarrier collision energies E.

The shape of a nucleus in highly excited states $i \geqslant 2$ can be spherical or deformed. The kind of surface deformation can be different for different high-energy states. The high-multipolarity $\lambda \geqslant 3$ axial or nonaxial multipole $\lambda \geqslant 2$ nuclear surface deformations usually lead to the smaller reduction of the barrier than those induced by the axial quadrupole surface deformation. Therefore, we can approximate $\sigma_i(E)|_{i\geqslant 2} \approx \sigma_0(E)$ and apply our model to high stellar temperatures $kT \lesssim 1$ MeV. As a result, the α-capture reaction cross section in star matter (3) can be rewritten as

$$\sigma(E, kT) \approx \left[P(0,0,kT) + \sum_{i=2}^{\infty} P(\varepsilon_i, j_i, kT)\right]\sigma_0(E) + P(\varepsilon_{\mathrm{vib}}, 2, kT)\sigma_1(E). \qquad (26)$$

Using the identity

$$\sum_{i=0}^{\infty} P(\varepsilon_i, j_i, kT) \equiv \left[P(0,0,kT) + \sum_{i=2}^{\infty} P(\varepsilon_i, j_i, kT)\right] + P(\varepsilon_{\mathrm{vib}}, 2, kT) \equiv 1, \qquad (27)$$

we reduce Eq. (26) to the simple form

$$\begin{aligned}\sigma(E, kT) &\approx \sigma_0(E) + P(\varepsilon_{\mathrm{vib}}, 2, kT)[\sigma_1(E) - \sigma_0(E)] \\ &= \sigma_0(E)\{1 + P(\varepsilon_{\mathrm{vib}}, 2, kT)[s(E) - 1]\}. \end{aligned} \qquad (28)$$

Here, the term containing $P(\varepsilon_{\mathrm{vib}}, 2, kT)$ is related to the cross-section enhancement induced by the population of the first 2^+ surface oscillation state in soft nuclei in the stellar matter, and

$$s(E) = \sigma_1(E)/\sigma_0(E). \qquad (29)$$

Ratio $s(E)$ directly shows the effect of cross-section enhancement caused by deformation of the nuclear surface in 2^+ states, because if the surface deformation is neglected, then $\sigma_1(E) = \sigma_0(E)$ and $s(E) = 1$.

If the nuclear surface is spherical in the ground and excited states, then $\sigma_i(E) = \sigma_0(E)$ for any i and $s(E) = 1$. In this case, we obtain from Eq. (3) using the identity (27) that

$$\sigma_{\mathrm{sph}}(E, kT) = \sigma_0(E). \qquad (30)$$

We can also obtain this result from Eq. (28) in the limit $s(E) = 1$.

The reaction S factor is proportional to the cross section [6] such that

$$S(E, kT) = E\exp(-2\pi\eta(E))\sigma(E, kT), \qquad (31)$$

where $\eta(E) = zZe^2/(\hbar v)$ is the Sommerfeld parameter, and $v = (2E/\mu)^{1/2}$ is the relative velocity in the entrance channel.

The enhancement of the S factor or the reaction cross section in stellar matter induced by the 2^+ surface oscillation is described by the ratio

$$\begin{aligned}s(E, kT) &= \frac{S(E, kT)}{S_{\mathrm{sph}}(E)} = \frac{\sigma(E, kT)}{\sigma_{\mathrm{sph}}(E, kT)} \\ &\approx 1 + P(\varepsilon_{\mathrm{vib}}, 2, kT)[s(E) - 1], \end{aligned} \qquad (32)$$

where

$$\begin{aligned}S_{\mathrm{sph}}(E) &= E\exp(-2\pi\eta(E))\sigma_{\mathrm{sph}}(E, kT) \\ &= E\exp(-2\pi\eta(E))\sigma_0(E). \end{aligned} \qquad (33)$$

The results of the S-factor enhancement for capture reactions $\alpha + {}^{52}\mathrm{Fe}$ and $\alpha + {}^{80}\mathrm{Sr}$ evaluated for star temperatures $kT = 0.2$ and 0.3 MeV are presented in Fig. 2. These temperatures are related to the O-Si burning phases in the core of massive stars [2,5]. As seen in this figure, the S factor is enhanced by the 2^+ surface oscillations at low collision energies E. The S-factor enhancement for reactions $\alpha + {}^{52}\mathrm{Fe}$ and $\alpha + {}^{80}\mathrm{Sr}$ grows with the temperature of the stellar matter in Fig. 2, because the partition of the 2^+ state increases in this temperature interval, see Fig. 1.

We also present results for the cross-section ratio $s(E)$ in Fig. 2. We see that values of $s(E)$ are significantly larger than 1 for both reactions at low (subbarrier) collision energies E. The cross-section enhancement induced by deformation is more important for the $\alpha + {}^{80}\mathrm{Sr}$ system.

The dependence of the transmission coefficient on the surface deformation is negligible at collision energies above the barrier. Therefore, $s(E, kT)$ and $s(E)$ are close to 1 at high collision energies, see Fig. 2.

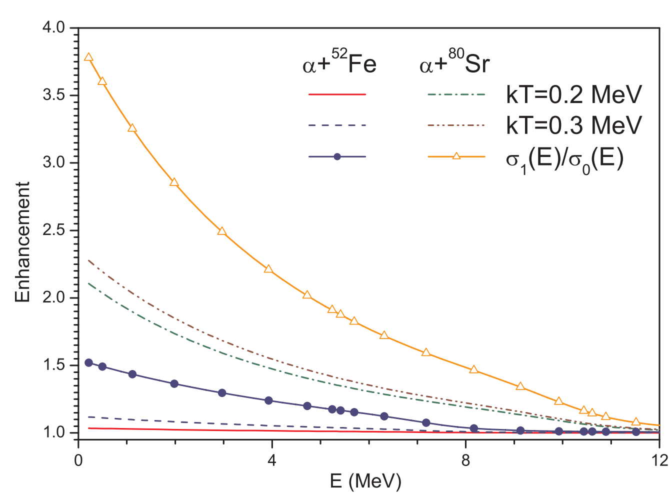

FIG. 2. (Color online) S-factor enhancement for capture reactions $\alpha + {}^{52}\mathrm{Fe}$ and $\alpha + {}^{80}\mathrm{Sr}$.

The stellar reaction cross sections are often averaged over the Maxwell-Boltzmann distribution of collision velocities v [2,12,13]. This cross section can be presented in the form [12]

$$\langle\sigma(kT)\rangle = \frac{2}{\sqrt{\pi}} \frac{\int_0^\infty \sigma(E,kT)E\exp(-E/kT)dE}{\int_0^\infty E\exp(-E/kT)dE}, \quad (34)$$

where brackets $\langle\ \rangle$ mean the Maxwell-Boltzmann averaging over the collision velocities. We determine the velocity-averaged cross sections $\langle\sigma_{\rm sph}(kT)\rangle$, $\langle\sigma_0(kT)\rangle$, and $\langle\sigma_1(kT)\rangle$ in the similar way. Note that $\langle\sigma_{\rm sph}(kT)\rangle = \langle\sigma_0(kT)\rangle$ due to Eq. (30).

The enhancement of velocity-averaged cross sections $\langle\sigma(kT)\rangle/\langle\sigma_{\rm sph}(kT)\rangle = \langle\sigma(kT)\rangle/\langle\sigma_0(kT)\rangle$ for α-capture reactions $\alpha + {}^{52}$Fe and $\alpha + {}^{80}$Sr caused by 2^+ shape vibrations is shown in Fig. 3. Ratios of velocity-averaged α-capture cross sections by nucleus with vibrating and spherical surfaces $\langle\sigma_1(kT)\rangle/\langle\sigma_0(kT)\rangle$ are also presented in this figure for comparison. As we see, the surface oscillations enhance the velocity-averaged cross sections. Because of this, the ratio $\langle\sigma(kT)\rangle/\langle\sigma_0(kT)\rangle$ is noticeably greater than 1 for $kT \gtrsim 0.1$ MeV. However, for low temperatures of star matter $kT \lesssim 0.1$ MeV, the partition probability of the 2^+ surface oscillation state is very small (see in Fig. 1); therefore, $\langle\sigma(kT)\rangle/\langle\sigma_0(kT)\rangle$ is close to 1. At star matter temperatures $kT \gtrsim 0.6$ MeV, the main contribution to the velocity-averaged capture cross section is related to a higher collision energy E, which is close to the barrier. Therefore, the effect of the barrier penetration enhancement due to surface vibrations of the nucleus in the 2^+ state is diminished.

When the results presented in Figs. 2 and 3 are compared, it is apparent that the 2^+ surface vibrations are important for both the S-factor values at low energies and the velocity-averaged cross sections at moderate star temperatures. The enhancement of the reaction S factor or the capture cross section induced by 2^+ nuclear surface excitation in stellar matter can change the balance between various reactions. This may affect the equilibrium conditions between direct and inverse reactions as well as the abundance of elements in hot stellar matter.

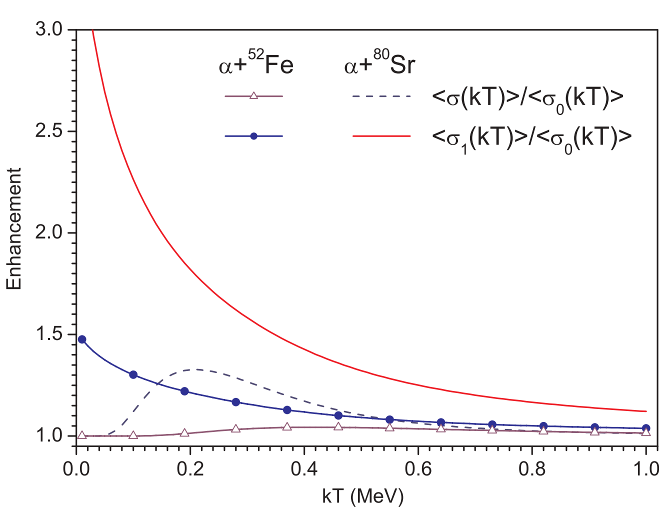

FIG. 3. (Color online) Enhancement of velocity-averaged cross sections of α-capture reactions $\alpha + {}^{52}$Fe and $\alpha + {}^{80}$Sr.

IV. *p*-CAPTURE REACTIONS IN STARS

We consider p-capture reactions on either a spherical or deformed nucleus in hot star matter using a formalism similar to that used for the α-capture reactions in the previous section. The proton spin is $j_p = \frac{1}{2}$. As a result, the nuclear part of the p-nucleus potential $v_n(r,\theta)$ should include the spin-orbit contribution [29,30] in contrast to the α-particle case. Therefore, we should modify the approach for cross-section evaluation presented in the previous section:

(i) The nuclear part of the α-nucleus potential described by Eqs. (16)–(22) is substituted by the Chepurnov p-nucleus potential with the central and spin-orbit parts [29]. The central part of the Chepurnov p-nucleus potential has also the Woods-Saxon shape (16), while its spin-orbit part is proportional to the radial derivative of the Woods-Saxon potential.

(ii) The deformation of the nuclear surface is also taken into account in the spin-orbit potential.

(iii) Summation over J in Eq. (4)

$$\frac{1}{2j_p+1}\sum_{J\ell\ell'}(2J+1)t_{J\ell\ell'}(E,\ell,\theta) = \frac{1}{2}\sum_{J\ell\ell'}(2J+1)t_{J\ell\ell'}(E,\ell,\theta) \quad (35)$$

is replaced by (see Refs. [18,30])

$$\sum_\ell[(\ell+1)\,t_{J=\ell+\frac{1}{2}}(E,\ell,\theta) + \ell\,t_{J=\ell-\frac{1}{2}}(E,\ell,\theta)], \quad (36)$$

where $t_J(E,\ell,\theta)$ is the transmission coefficient, which shows the probability of proton penetration through the barrier formed at the angle θ between the symmetry axis of the axial-symmetric deformed nucleus and the vector directed from the center of the deformed nucleus to the proton, and J is the total angular momentum $\vec{J} = \vec{\ell} + \vec{j}_p$.

Note that Eq. (36) is exact in the case of p capture on a nucleus in the state with zero value of the spin (see Appendix A in Ref. [18]). We would like to roughly estimate the deformation effect on p capture. Therefore, for the sake of simplicity, we neglect the spin of nuclear states in a heavy nucleus and apply Eq. (36) to p capture on the nucleus with any value of spin.

From Fig. 4, we can see that the 2^+ surface oscillations weakly affect the S-factor values for reactions $p + {}^{52}$Fe and $p + {}^{80}$Sr. The enhancement of p-capture cross sections averaged over the Maxwell-Boltzmann distribution of collision velocities induced by surface vibrations for these reactions are given in Fig. 5. As seen, the 2^+ surface oscillations slightly enhance the velocity-averaged cross sections for p-capture reactions.

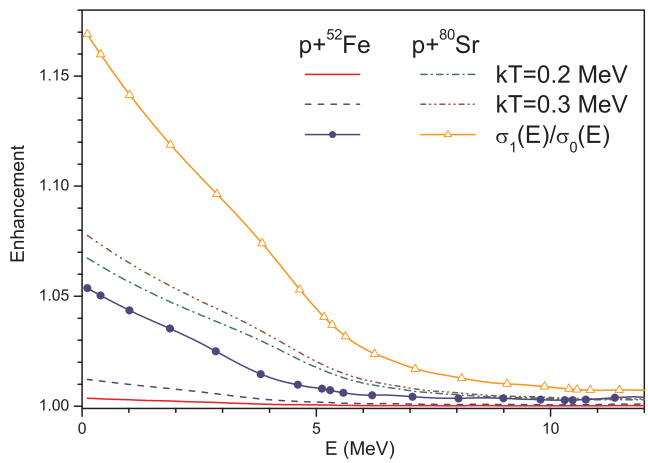

FIG. 4. (Color online) S factor enhancement for capture reactions $p + {}^{52}$Fe and $p + {}^{80}$Sr.

V. NUCLEUS-NUCLEUS FUSION REACTIONS IN STARS

Both nuclei participating in fusion reactions can be excited in hot stellar matter. Therefore, the nucleus-nucleus fusion cross sections can be evaluated as

$$\sigma(E,kT) = \sum_{i_1,i_2=0}^{\infty} P_1(\varepsilon_{i_1}, j_{i_1}, kT) \times P_2(\varepsilon_{i_2}, j_{i_2}, kT)\sigma_{i_1 i_2}(E), \qquad (37)$$

where $P_1(\varepsilon_{i_1}, j_{i_1}, kT)$ and $P_2(\varepsilon_{i_2}, j_{i_2}, kT)$ are the partition probabilities of states i_1 in the first nucleus and i_2 in the second one, respectively, and $\sigma_{i_1 i_2}(E)$ is the fusion reaction cross section between these nuclei in the states i_1 and i_2.

If one nucleus is rigid while another one is soft, the 2^+ surface oscillations are mainly important in the soft reaction partner. In that case, the double sum in Eq. (37) reduces to the single sum, see Eq. (2). Therefore, we can easily adapt our approach for the cross-section evaluation presented in previous sections to the nucleus-nucleus fusion reactions.

Let us consider fusion reaction ^{16}O+^{32}Mg. This reaction may take place in massive stars during explosive O-Si burning phases. The neutron-reach ^{32}Mg can be formed in hot star matter by n capture. The compound nucleus formed in this reaction is ^{48}Ca.

Nucleus ^{16}O is spherical, double magic, and very rigid. The energies of the lowest 0^+, 3^-, and 2^+ states in ^{16}O are, respectively, 6.05, 6.13, and 6.92 MeV [17,26]. Therefore, probabilities of population of such high-energy states in stars at $kT \lesssim 1$ MeV are negligible. The ground-state shape of ^{32}Mg is spherical [31]. The excitation energy of the lowest 2^+ surface oscillation state in ^{32}Mg is $\varepsilon_{\rm vib} = 0.8855$ MeV [17]. The total vibrational amplitude of this state is very large, $\beta_{\rm vib} = 0.473$ [17]. (Data for other exited states are picked up from Ref. [26].) So, the neutron-reach nucleus ^{32}Mg is very soft. In this case, the fusion reaction model described in Sec. III may be applied, with minor modifications, to the reaction ^{16}O+^{32}Mg. The modifications are only related to the nuclear part of the interaction potential.

The functional form of the nuclear interaction potential between spherical nuclei has been discussed in Ref. [25]. This potential is obtained by using the semimicroscopic calculations of the interaction energy of two nuclei. Heights and radii of empirical fusion barriers are well described by this potential for various pairs of interacting nuclei [25,32,33].

The nuclear part of the potential depends on both the distance between surfaces of colliding nuclei $d(r,\theta)$ and the surface curvature of corresponding nuclei at the closest point [34]. Therefore, the nuclear part of the potential between spherical and deformed nuclei may be approximated as

$$v_n(d(r,\theta)) \approx (C_1 + C_2)/(C_1 + C_2(\theta))\, v_n^0(d(r,\theta)). \qquad (38)$$

Here, $v_n^0(d(r))$ is the nuclear part of the interaction potential between two spherical nuclei with radii R_1^0 and R_2^0 at a distance between their surfaces $d(r) = r - R_1^0 - R_2^0$ [25]; $C_{1(2)} = 1/R_{1(2)}^0$ are the surface curvatures of corresponding spherical nuclei; $R_2(\theta) = R_2^0(1 + \beta Y_{20}(\theta))$ is the distance of the surface from the origin of the deformed nucleus; $C_2(\theta) \approx (1 + 2\beta Y_{20}(\theta))/R_2^0$ is the curvature of the deformed nuclear surface; and $d(r,\theta) \approx r - R_1^0 - R_2(\theta)$. Substituting potential (38) into Eq. (13), we can estimate the cross section of the heavy-ion fusion reaction in the framework of our model.

The S-factor enhancement for fusion reaction ^{16}O+^{32}Mg induced by the 2^+ surface oscillation in ^{32}Mg is shown in Fig. 6. The S-factor values are strongly enhanced by the surface oscillations, especially at low collision energies E. The enhancement of the fusion cross section for this reaction averaged over the Maxwell-Boltzmann distribution of collision velocities is presented in Fig. 7. As seen there, the 2^+ surface oscillations increase the velocity-averaged cross sections of nucleus-nucleus fusion reactions in stellar matter.

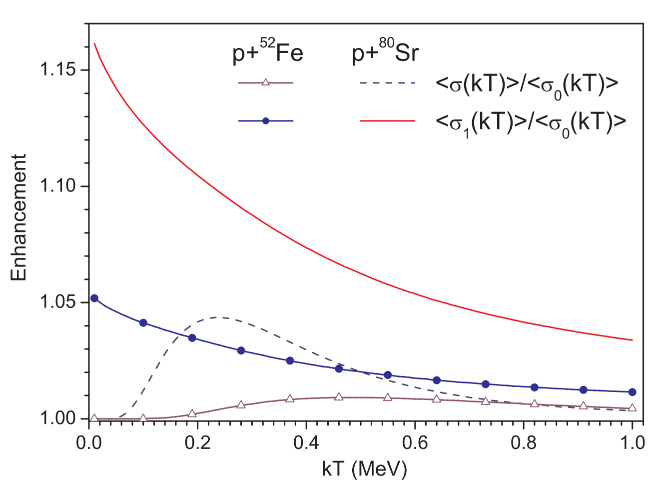

FIG. 5. (Color online) Enhancement of velocity-averaged cross sections of p-capture reactions $p + {}^{52}$Fe and $p + {}^{80}$Sr.

VI. CAPTURE OF CHARGED PARTICLES ON NUCLEI WITH WELL-DEFORMED GROUND STATE IN STARS

If a nucleus is well deformed in the ground state, then two kinds of surface deformations (the static one of the ground state and the dynamic one of the 2^+ vibrational state) should be taken into account when evaluating the interaction potential between a charged particle and the nucleus. The dynamic

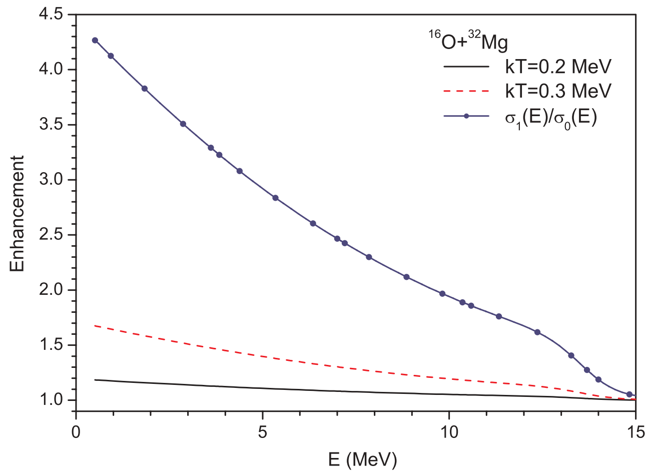

FIG. 6. (Color online) S-factor enhancement for fusion reaction $^{16}O+^{32}Mg$.

deformations are related to quadrupole surface vibrations about the deformed ground-state shape of the nucleus. Both types of quadrupole surface deformations should affect the cross sections of nuclear reactions in hot stellar matter.

We can extend our model for evaluating capture reactions to a nucleus with a deformed ground-state shape. The extension to such a case is straightforward. The static amplitude β_{static} should be taken into account in the evaluation of capture on the nucleus in the ground state $\sigma_0(E)$, whereas the static and dynamic deformation amplitudes should be summed up for estimating capture on the nucleus in the first 2^+ vibrational state $\sigma_1(E)$. By putting $D(\beta) = \delta(\beta - \beta_{\text{static}})$, we adapt Eqs. (7), (10), and (12) to obtain $\sigma_0(E)$, see also Ref. [23]. The influence of the 2^+ surface oscillations around the deformed ground-state shape on physical quantities will be similar to the case of the 2^+ surface vibrations around the spherical shape. Thus, we should substitute $D(\beta)$ in Eq. (10) by $D(\beta - \beta_{\text{static}})$ to estimate the effect of the 2^+ surface oscillations around the deformed shape. Here, we take into account the fact that vibrations of β type about the deformed shape [27] have the largest effect on the particle-nucleus potential and that the natures of quadrupole surface oscillations in deformed and spherical nuclei are similar.

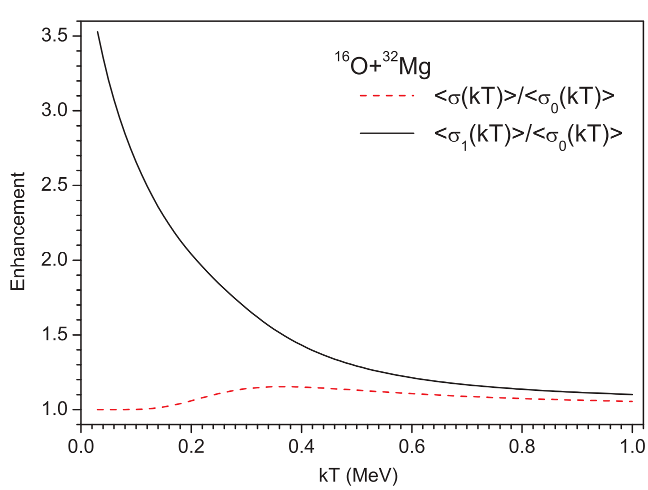

FIG. 7. (Color online) Enhancement of velocity-averaged cross section for reaction $^{16}O+^{32}Mg$.

A. α capture

Reactions $\alpha + ^{22}$Ne and $\alpha + ^{24}$Mg are very important in the burning of massive stars and nucleosynthesis in stellar matter [5,8].

Nuclei ^{22}Na and ^{24}Mg are well deformed in the ground state. The values of the corresponding deformation parameters are $\beta_{\text{static}} = 0.326$ and 0.374 [31]. Excitation energies of the first 2^+ state in these nuclei are, respectively, 1.27 and 1.37 MeV [17]. However, the nature of these excited states is rotational. Energies of the lowest 2^+ vibrational state are, correspondingly, 4.46 and 4.24 MeV [26]. Energies are sufficiently high; therefore, the occupation probabilities of the lowest 2^+ vibrational excitation in ^{22}Na or ^{24}Mg in stellar matter at temperatures $kT \lesssim 0.3$ MeV are small. Consequently, we can ignore the dynamic deformations related to the 2^+ vibrational state in these nuclei and take into account only the static deformation. Shapes of the nucleus in the ground and low-spin rotational excited states are very similar; therefore, the cross sections for these states are the same. Occupation probabilities of the ground and low-spin rotational excited states are summed, and the net probability of these states is very close to 1.

We present the evaluation of the ratio $\sigma_{\text{static}}(E)/\sigma_{\text{sph}}(E)$ for reactions $\alpha + ^{22}$Ne and $\alpha + ^{24}$Mg in Fig. 8, where $\sigma_{\text{static}}(E)$ and $\sigma_{\text{sph}}(E)$ are the cross sections obtained, respectively, for deformed and spherical shapes of heavy nuclei. We see that the enhancement of the cross sections or the S factors and the velocity-averaged cross sections for reactions $\alpha + ^{22}$Ne and $\alpha + ^{24}$Mg induced by static deformation is strong. This enhancement increases as the temperature of stellar matter decreases.

Comparing results in Fig. 8, we conclude that enhancement of the capture cross section caused by static deformation of Mg is larger than that for the Ne case. There are two reasons for

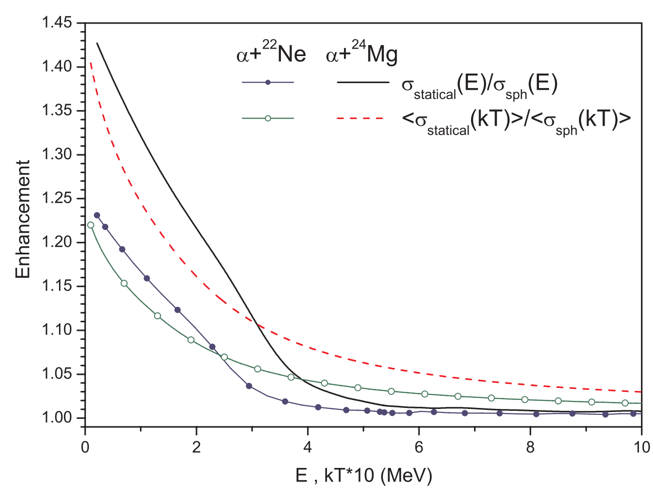

FIG. 8. (Color online) Enhancement of S factors and velocity-averaged cross sections for reactions $\alpha + ^{22}$Ne and $\alpha + ^{24}$Mg induced by static deformation of heavy nuclei.

such an effect. First, the value of the deformation parameter in ^{24}Mg is larger than that in ^{22}Ne. Second, the number of protons in ^{24}Mg is also larger than that in ^{22}Ne.

The range of barrier distribution (24) or (25) for the α-capture reaction is induced both by both the static deformation of the heavy nucleus β and by the mutual orientation of the heavy nucleus and the incoming α particle (angle θ). In contrast to this, the treatment of α-capture reactions on deformed nuclei based on an effective spherical potential of equal volume is related to a single barrier. The height of this single barrier $v_{\rm bar}^{\rm sph}(\beta)$ estimated at deformation β is slightly smaller than that evaluated for the same colliding system proceeding from systematics for the α-nucleus potential $v_{\rm bar}^{\rm sph.\ syst.}$. The barrier reduction due to volume correction is proportional to β^2 for effective spherical potential of equal volume, while variation of the barrier induced by β in our approach [see Eqs. (14)–(22)] is proportional to β and depends on θ. As a result, the values of potentials obey the inequalities

$$v_{\rm bar}(\theta = 0)|_{\beta>0} < v_{\rm bar}^{\rm sph}(\beta) < v_{\rm bar}^{\rm sph.\ syst.} < v_{\rm bar}\left(\theta = \frac{\pi}{2}\right)\Big|_{\beta>0}.$$

Therefore, the transmission coefficients $t(E, \ell, \theta)$ and the ratio $\sigma_{\rm static}(E)/\sigma_{\rm sph}(E)$ are differently described in the frameworks of these two approaches at very low collision energies E. The difference between cross-section values evaluated in both approaches increases with the value of the deformation parameter.

B. *p* capture

The p-capture reactions on proton-rich nuclei are important for rp-nucleosynthesis in stellar matter [2,7]. Therefore, let us consider the reactions $p + {}^{72,76}$Kr. The ^{72}Kr is a significant waiting point in the rp-process [7]. Proton-rich nuclei ^{72,76}Kr are well-deformed in the ground state. It is interesting that the ground-state shape is oblate for ^{72}Kr and prolate for ^{76}Kr. Values of the corresponding deformation parameter are $\beta_{\rm static} = -0.349$ and 0.4 [31].

The p-capture reaction below the barrier is also strongly enhanced by the ground-state deformation of nuclei, see Fig. 9.

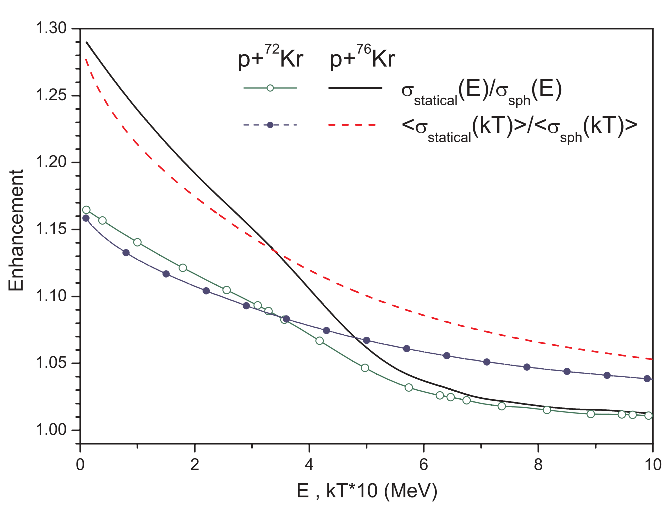

FIG. 9. (Color online) Same as Fig. 8, but for reactions $p + {}^{72,76}$Kr.

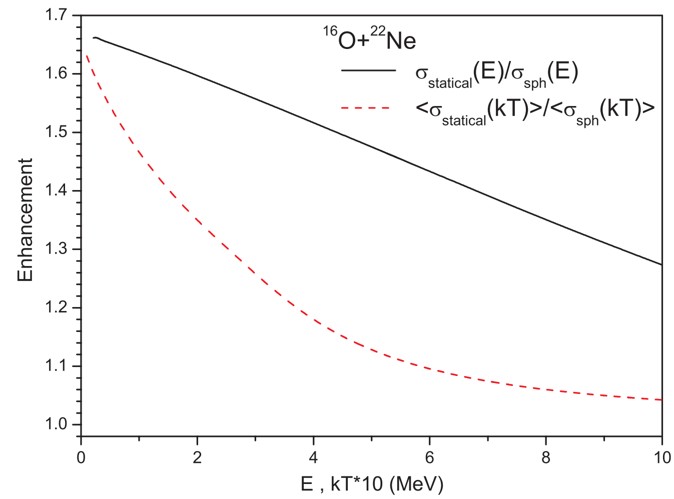

FIG. 10. (Color online) Same as Fig. 8, but for ^{16}O+^{22}Ne.

We neglect the dynamic deformation related to the 2^+ vibrational state in ^{72,76}Kr, because energies of the lowest vibrational states in these nuclei are much larger than those of the lowest rotational levels [26]. Because of this, vibrational levels in these nuclei are weakly populated in stellar matter. The proton spin is exactly taken into account in our evaluation in this case, because the ground-state spin of even-even nuclei ^{72,76}Kr is zero and we neglect the dynamic deformations. Comparing the results in Fig. 9 reveals that the prolate deformation enhances the p-capture reaction cross section more strongly than the similar oblate deformation.

C. Heavy-ion fusion

The enhancement of the S factor and velocity-averaged cross section for the ^{16}O+^{22}Ne reaction induced by static deformation of the heavy nucleus is presented in Fig. 10. The properties of excited states in the colliding nuclei were discussed before. Therefore, we take into account only the static ground-state deformation of ^{22}Ne in our calculations. Reaction ^{16}O+^{22}Ne may take place at the O-Si burning phase in hot stellar matter. We see very strong enhancement of both the S factor and the velocity-averaged cross section, especially at low temperatures of stellar matter.

VII. DISCUSSION AND CONCLUSIONS

We demonstrated that the fusion (capture) reactions between two nuclei or between a charged particle and a nucleus at subbarrier collision energies are enhanced by the 2^+ surface oscillations in soft nuclei in hot stellar matter. The cross sections averaged over collision velocities are also increased by the 2^+ surface vibrations. The effect of cross-section enhancement, induced by the shape oscillations, depends on both the temperature of the stellar matter and the excitation energy of 2^+ state.

Comparing the results for α-, p-capture reactions and heavy-ion fusion reactions indicates that the influence of the 2^+ surface vibrations grows with increasing charges and masses of the colliding particles. This influence is negligible for the p-capture and important for the α-capture and heavy-ion fusion reactions.

The enhancement of capture cross sections by quadrupole surface oscillations is significant for soft nuclei with low-energy 2^+ states. Such nuclei often occur far from the β-stability line. This enhancement of capture rate is negligible in the case of rigid nuclei. Therefore, the mass and charge dependencies of the lowest 2^+ state are significant for abundances of heavy proton-rich elements in hot star matter [7,8]. Reactions with heavier colliding partners are more strongly enhanced by the low-energy 2^+ oscillations, because the barrier reductions induced by axial quadrupole oscillations of nuclear surface are larger in heavier systems.

Our model can be easily extended to the case of high-multipole $\lambda \geqslant 3$ axial surface oscillations. However, the energies of high-multipole surface vibrations are higher than those for $\lambda = 2$ in most nuclei [17]. Moreover, the amplitudes of the potential barrier reductions due to axial $\lambda \geqslant 3$ or nonaxial $\lambda \geqslant 2$ multipole surface distortions are smaller than those caused by the axial quadrupole deformation. So the effect of high-multipole axial or nonaxial nuclear surface vibrations may be significant only for the charged-particle capture on nuclei, which are especially soft to a specific kind of surface deformation.

We have studied the enhancement of fusion reaction cross sections induced by the 2^+ surface oscillations of nuclei participating in reactions in star matter. However, many other mechanisms of subbarrier fusion enhancement have been discussed, see Refs. [14,19,21,22,28,32] and references therein. Therefore, it is interesting to evaluate the role of various reaction mechanisms in the framework of detailed models.

The enhancement of reaction cross sections, induced by the surface oscillations of colliding nuclei, is important for careful evaluation of the equilibrium compositions of nuclei in hot stellar matter. The 2^+ surface oscillations in soft nuclei with spherical ground-state shape should be considered in the description of nucleosynthesis during supernova explosions.

It is shown that p and α captures on nuclei with well-deformed ground states are significantly enhanced by the ground-state deformation of nuclei at low temperatures of stellar matter. Similar effects also take place in fusion reactions between deformed heavy ions. This effect is also very important in the O-Si burning phases of massive stars, when reactions between charged particles and well-deformed nuclei such as $^{19-21}$F, $^{20-24}$Ne, $^{20-26}$Ne, $^{22-28}$Mg, $^{22-29}$Al, and $^{24-29}$Si determine the burning processes in massive stars. Enhancement of the charged-particle capture reactions by the ground-state deformation is very significant for the nucleosynthesis of proton-rich elements in hot stellar matter. The origin of the large abundances of proton-rich nuclei with well-deformed shapes, especially in the $Z = 36$–42 region [7], may be associated with enhancement of the charged-particle capture cross section caused by the ground-state deformation.

Comparing results for the velocity-averaged cross sections in Figs. 3, 5, 7, and 8–10, we see that the enhancement of the reaction cross section caused by static deformation is very strong for low temperatures of star matter. However, the deformation effect for reactions with soft spherical nuclei is important at high temperatures, when the occupation probability of the lowest 2^+ vibrational state is significant. The enhancement of the reaction cross section related to the static deformation is very strong at low temperature, because the ground state is mainly populated at low temperatures.

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