Isoscalar transverse oscillations of the current in the Landau zero-sound approximation

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(Submitted 20 June 1984)
Yad. Fiz. **42**, 351–354 (August 1985)

With the help of the exact distribution function of zero sound it is shown that in the Landau zero-sound approximation with a spin-independent interaction, isoscalar transverse twist oscillations of the current are impossible. The reason for the wrong conclusion of several recent articles about the existence of such oscillations is the unjustified truncation of the expansion in the p momenta of the distribution function of the zero sound.

1. In the Landau theory of Fermi liquids[1–3] for the interaction that does not depend on the spins of the quasiparticles, one has noted the possibility of transverse twist oscillations of the current without variation of the density. For nuclei such current oscillations were first studied by Holzwarth and Eckart in the hydrodynamical approximation,[4,5] which was derived in Refs. 6 and 7 from the Landau kinetic equation for the zero sound by expansion of the exact distribution function of the zero sound with respect to the multipolarity of the deformation of the Fermi surface and truncation of the resulting series at the quadrupole deformation. However, as will be shown below with the help of the exact distribution function of the zero sound, such oscillations of the current are impossible.

2. Direct extension of the Landau theory,[1–3] which was formulated for an infinite isotropic medium, to nuclei is difficult because of their finite size. However, the sufficiently abrupt change of the density on the nuclear boundary allows one to define an effective nuclear surface,[8,9] which considerably simplifies the solution of the problem. Inside the nucleus the zero-sound oscillations of nuclear matter are described by the Landau equation for the zero sound in an infinite medium. On the effective surface of the nucleus these oscillations satisfy certain boundary conditions.[9,10]

Let us write down the equation for the distribution function of the quasiparticles inside the nucleus at zero temperature[1–3,10]:

$$\frac{\partial f^{(0)}(\mathbf{r},\mathbf{p},t)}{\partial t}+\mathbf{v}\frac{\partial}{\partial \mathbf{r}}\Big[f^{(0)}(\mathbf{r},\mathbf{p},t)+\delta(\varepsilon-\varepsilon_F)\times\frac{2\pi^2\hbar^3}{p_F M^*}\int\frac{d\mathbf{p}}{(2\pi\hbar)^3}F(\mathbf{p},\mathbf{p}')f^{(0)}(\mathbf{r},\mathbf{p}',t)\Big]=0. \qquad (1)$$

Here $\mathbf{v}=v_F(\mathbf{p}/p_F)$ is the velocity of the quasiparticles on the Fermi surface, $v_F=(2\varepsilon_F/M^*)^{1/2}$, M^* is the effective mass of the quasiparticles, p_F is the Fermi momentum, ε_F is the Fermi energy, and $F(\mathbf{p},\mathbf{p}')$ is a constant in the amplitude of the quasiparticle interaction expressed in the units $2\pi^2\hbar^3/p_F M^*$.

A particular solution of (1), which corresponds to the propagation of a plane wave, has the form[1]

$$f_{\mathbf{k}}^{(0)}(\mathbf{r},\mathbf{p},t)=\delta(\varepsilon-\varepsilon_F)\nu(\mathbf{p},\mathbf{k})\exp(i(\mathbf{kr}-\omega t)), \qquad (2)$$

where $\mathbf{k}$ is the wave vector in the direction of propagation of the plane wave. Since according to (2) the quantity $\mathbf{p}$ is fixed, $F(\mathbf{p},\mathbf{p}')$ depends only on the angle between $\mathbf{p}$ and $\mathbf{p}'$. In the standard expansion

$$F(\mathbf{p},\mathbf{p}')=F_0+F_1\cos(\mathbf{p},\mathbf{p}')+\dots,$$

we retain only the two components mentioned above. When $F_1\neq 0$,

$$M^*/M=1+F_1/3$$

where M is the nucleon mass,[1–3] and (1) has three branches of solutions (2) corresponding to the values $m=0$ and $m=\pm 1$ of the summation index in the expansion

$$\cos(\mathbf{p},\mathbf{p}')=(4\pi/3)\sum_m Y^*_{1m}(\mathbf{p},\mathbf{k})\,Y_{1m}(\mathbf{p}',\mathbf{k}).$$

The $m=0$ branch, which corresponds to density oscillations, was studied in detail in Ref. 10. The branches $m=\pm 1$, which are considered here, correspond to the oscillations of the current without change of the density. In this case the function $\nu(\mathbf{p},\mathbf{k})$ in (2) has the form[1,2]

$$\nu(\mathbf{p},\mathbf{k})=\frac{\cos(\theta_{pk})\sin(\theta_{pk})}{s-\cos(\theta_{pk})}(\alpha_1 e^{i\varphi_{pk}}+\alpha_{-1}e^{-i\varphi_{pk}}). \qquad (3)$$

Here α_1 and α_{-1} are constants and s is the ratio of the velocity $v^{(0)}$ of the zero sound of v_F. The quantity s is related to the constant F_1 in the interaction amplitude of the quasiparticles by the equation

$$G(s)=(s/2)\ln((s+1)/(s-1))-1=(F_1-6)/(3F_1(s^2-1)), \qquad (4)$$

which has real $s>1$ solutions only when $F_1>6$. The frequency of the oscillations is

$$\omega^{(0)}=k^{(0)}v^{(0)}=sk^{(0)}v_F. \qquad (5)$$

For an infinite medium the solution (2)–(5) is a physical one. Since (1) is a linear homogeneous equation, for a finite system one can construct a solution by taking a superposition of the plane-wave solutions (2) in the form

$$f^{(0)}(\mathbf{r},\mathbf{p},t)=\int d\mathbf{k}A(\mathbf{k},\mathbf{z})f_{\mathbf{k}}^{(0)}(\mathbf{r},\mathbf{p},t). \qquad (6)$$

Here $A(\mathbf{k},\mathbf{z})$ is a weight function and $\mathbf{z}$ is the preferred direction in space. We are interested in the oscillations with fixed frequency; therefore, because of the relation between ω and k in the dispersion relation (4) the integration in (6) will be performed only over the angles of the vector $\mathbf{k}$.

The density oscillations are

$$\rho^{(0)}(\mathbf{r},t)=2\int\frac{d\mathbf{p}}{(2\pi\hbar)^3}f^{(0)}(\mathbf{r},\mathbf{p},t). \quad (7)$$

Substituting (2), (3), and (6) into (7), we find that $\rho^{(0)}(\mathbf{r},t)$ vanishes for any weight function $A(\mathbf{k},\mathbf{z})$.

Integrating the equation (1) over $\mathbf{p}$, one obtains the continuity equation, which in the absence of density oscillations has the form

$$\operatorname{div}\mathbf{j}^{(0)}(\mathbf{r},t)=0, \quad (8)$$

with the current in the form

$$\mathbf{j}^{(0)}(\mathbf{r},t)=\frac{2}{M}\int\frac{d\mathbf{p}}{(2\pi\hbar)^3}\mathbf{p}f^{(0)}(\mathbf{r},\mathbf{p},t). \quad (9)$$

From (8) it follows that for any weight function $A(\mathbf{k},\mathbf{z})$ the current $\mathbf{j}^{(0)}(\mathbf{r},t)$ has a twist character.

Let us require that the current oscillations have an angular dependence of a magnetic type[1)]:

$$\mathbf{j}^{(0)}(\mathbf{r},t)=R(r)\,[\mathbf{Y}^{(0)}_{1,+1}(\mathbf{r},\mathbf{z})-\mathbf{Y}^{(0)}_{1,-1}(\mathbf{r},\mathbf{z})]\exp(-i\omega t), \quad (10)$$

where $R(r)$ is a certain function and $\mathbf{Y}^{(0)}_{1,\pm1}(\mathbf{r},\mathbf{z})$ are the spherical vectors of the magnetic type.[11] For convenience we have taken the time dependence in the form $\exp(-i\omega t)$. Substituting (6) into (9), taking into account that (9) satisfies the Helmholtz equations and Eq. (8), and making use of the completeness and orthogonality of the spherical vectors, we obtain

$$R(r)=\alpha\frac{32\pi^2p_F^2(M^*/M)\,i}{(2\pi\hbar)^3}\sqrt{\frac{4\pi}{3}}\,j_1(kr)\left((1-s^2)G(s)+\frac{1}{3}\right), \quad (11)$$

$$\alpha_1=-\alpha_{-1}=-\alpha,\quad A(\mathbf{k},\mathbf{z})=1.$$

Here $j_1(x)$ is the Bessel spherical function. Going over to the system of coordinates rotated by the angle $\theta=\pi/2$ (the $\mathbf{z}'$ axis), we express the current (9), (10) in terms of a single spherical vector:

$$\mathbf{j}^{(0)}(\mathbf{r},t)=-\sqrt{2}R(r)\mathbf{Y}^{(0)}_{1,0}(\mathbf{r},\mathbf{z}'). \quad (12)$$

This expression is more convenient for the analysis. The current is related to the displacement in the following way:

$$\mathbf{j}^{(0)}(\mathbf{r},t)=\rho\frac{\partial}{\partial t}\boldsymbol{\delta}^{(0)}(\mathbf{r},t), \quad (13)$$

where ρ is the nucleon density in the nucleus. From (10) and (13) it follows that the radial components of the current and displacement are zero, i.e., the oscillations under consideration are transverse. For small displacements $\boldsymbol{\delta}^{(0)}(\mathbf{r},t)$ one can approximately define the twist angle $\Omega(r)$ of nuclear matter at a given point as a ratio of the magnitude of the displacement at this point to the distance of the point from the center of the nucleus. From (12) we find the radial dependence of the twist angle in the form

$$\Omega(r)=j_1(kr)/r. \quad (14)$$

In order to find the value of the wave vector k we shall use the following boundary condition on the effective surface of the nucleus suggested in Ref. 5:

$$\left.\frac{d\Omega(r)}{dr}\right|_{r=R}=\left.\frac{d}{dr}\left(\frac{j_1(kr)}{r}\right)\right|_{r=R}=0. \quad (15)$$

Knowing k from the boundary condition, for $F_1>6$ one can determine the oscillation frequency ω with the help of (5) and the dispersion relation (4). For the quasiparticles in the nucleus $|F_1|\lesssim 1$ (Ref. 12), and for such values of the constant the dispersion relation (4) has no solutions.

Let us now clarify the reason for examination of these oscillations in Refs. 4–7. To this end we expand the exact distribution function (2) in the multipolarity of the deformation of the Fermi surface, which corresponds to the representation of $\nu(\mathbf{p},\mathbf{k})$ in the form

$$\nu(\mathbf{p},\mathbf{k})=\sum_{lm}\nu_{lm}Y_{lm}(\mathbf{p},\mathbf{k}). \quad (16)$$

Following Refs. 6 and 7, we truncate this series at $l=2$, and, substituting the truncated series into the kinetic equation (1), find the relation between $\nu_{1,\pm1}$ and $\nu_{2,\pm1}$ and obtain the dispersion relation

$$s^2=(\omega^{(0)}/k^{(0)}v_F)^2=(1+F_1/3)/5. \quad (17)$$

Note that this dispersion relation has a solution for $F_1>-3$, while the dispersion relation (4), obtained with the exact distribution function of the zero sound, has a solution only when $F_1>6$. Let us calculate the current and the radial dependence of the twist angle with the help of the truncated distribution function of the zero sound.

After algebra analogous to (2)–(12), we calculate the current which has the same functional dependence on the coordinates as (12). Therefore, the radial dependence of the twist angle $\Omega(r)$ and the boundary condition coincide in the cases of the exact distribution function of the zero sound and the distribution function truncated at the quadrupole deformation of the Fermi surface, while the dispersion relations are significantly different. Setting $F_1=0$ in (17) for the first root of the equation (14), we find the oscillation frequency

$$\hbar\omega=\hbar(5.76/\sqrt{5}R)\sqrt{2\varepsilon_F/M}\simeq142.5\ (\text{fm}/R)\cdot\text{MeV}. \quad (18)$$

Here the values of the Fermi energy $\varepsilon_F=36.87$ MeV corresponds to the value $\rho=0.16$ fm of the equilibrium density, obtained in Ref. 5 with the help of the density-dependent effective forces. It is this value of the oscillation frequency that was obtained in Ref. 5.

Such a discrepancy between the results calculated with the exact distribution function of the zero sound and the function truncated at the quadrupole deformation of the Fermi surface arises because the dispersion relation (7) has a solution in the region $F_1\leqslant6$ of the values of the constant, while the exact dispersion relation (4) has no solutions in this region. The dispersion relation (17) was obtained by truncation of the distribution function of the zero sound at the quadrupole deformation of the Fermi surface. Therefore, for realistic values of the constant F_1 in the quasiparticle interaction amplitude the distribution function of the zero sound may not be truncated and the isoscalar transverse twist oscillations of the current are impossible.

To conclude, the author expresses his deep gratitude to V. M. Strutinskiĭ for numerous discussions of this work and to V. I. Abrosimov, A. G. Magner, and R. V. Hasse for useful comments.

[1)]Note that here we restrict ourselves the twist oscillations of the current whose angular dependence is described by the spherical vectors of the multipolarity $l = 1$. However, the main conclusion of the paper stems from the dispersion relations (4) and (17) (see below) and does hold for the $l > 1$ oscillations of the current, which have also been studied in Refs. 4 and 5.

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Translated by Gregory Toker