EMPIRICAL RELATIONS FOR α-DECAY HALF-LIVES

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The empirical relations for the half-lives of α-transitions between the ground states of parent and daughter nuclei with even/odd numbers of protons/neutrons are presented. Experimental data for half-lives of 344 α-radioactive isotopes are analyzed. The corresponding parameters for the whole collection of nuclei and separately for heavy (heavier than ^{208}Pb) and light nuclei are determined. The calculations are performed with regard for both the effect of screening of a nucleus by electrons and the nonzero orbital moment of an α-particle for α-decays of even-odd, odd-even, and odd-odd nuclei.

1. Introduction

The phenomenon of α-radioactivity has been studied already more than a century and remains topical till now in the frame of experimental sciences and in practical applications. Radioactivity is the first nuclear process observed by A.H. Becquerel (in 1896) [1], and α-rays are the least penetrating emission of radioactive nuclei. In 1909, E. Rutherford proved that α-particles are twice ionized atoms ^{4}He [2], and the first empirical formula describing the dependence of the half-life on the energy of α-particles was deduced already in 1911 (the Geiger–Nuttall relation)[1]. Till those times, the process of α-decay of nuclei is comprehensively and successfully studied within the microscopic and macroscopic-cluster approximations and within the fission theory. New empirical formulas connecting the half-life with the energy of an α-particle and the mass number and charge of the parent nucleus have been proposed in [3–10]. Some formulas describe the α-decay of all nuclei with a single function, whereas the other ones indicate the difference between the decays of even-even (e-e), even-odd (e-o), odd-even (o-e), and odd-odd (o-o) nuclei. However, mostly are considered the transitions between ground states of parent and daughter nuclei (even-even nuclei in the most cases) without regard for both the effect of screening of nuclei by electrons and a nonzero orbital moment of the α-transition.

By planning the experiment, in which the α-radioactivity is manifested or studied, it is necessary to evaluate the probable α-decay half-life for the corresponding nuclei, and the experimenters use most often simple empirical functions.

In our work, we present a collection of empirical relations describing the transitions from the ground state of e-e, e-o, o-e, and o-o parent nuclei to the ground sate of daughter ones. The calculations were performed for the whole set of nuclei (collection I), as well as for heavy/light nuclei (collection II/collection III) separately. The formulas for $T_{1/2}$ for e-o, o-e, and o-o nuclei contain the additional terms depending on the orbital moment of an α-particle. This orbital moment can be nonzero for the given transitions. In the calculations of the transition energy Q, we took the effect of screening of nuclei by electrons into account. The work is organized as follows: in Section 1, we discuss the input data; Section 2 presents the survey of the theoretical basis and the results of calculations; Section 3 contains conclusions.

2. Input Data

We have analyzed the experimental data on $T_{1/2}$ for 344 α-transitions between the ground state of parent and daughter nuclei (the collection of input data is identical to that in [11]). They include 136 even-even (e-e), 84 even-odd (e-o), 76 odd-even (o-e), and 48 odd-odd (o-o) α-radioactive isotopes; 200 light nuclei (A $\leq ^{206}$Pb) and 144 heavy ones. We used the reliably measured experimental values of the half-lives, spins, parities, and probabilities of the transitions between the ground state of nuclei [12, 13]. In calculations of the energies of reactions, we used values of the mass excesses of isotopes taken from [12].

3. Results

The quantity Q for α-decays is defined as follows:

$$Q = \Delta M_p - (\Delta M_d + \Delta M_\alpha) + k(Z_p^\epsilon - Z_d^\epsilon), \qquad (1)$$

where ΔM_p, ΔM_d, and ΔM_α are, respectively, the mass excesses of parent and daughter nuclei and an α-particle, $Z_{p(d)}$ is the charge of the parent (daughter) nucleus. The last term $kZ^\epsilon_{p(d)}$ describes the effect of screening of the nucleus by electrons. Here, $k = 8.7$ eV and $\epsilon = 2.517$

for nuclei with $Z \geq 60$, and $k = 13.6$ eV and $\epsilon = 2.408$ для nuclei with $Z < 60$ [3]. This term was introduced in order to directly describe the interaction of an α-particle with a nucleus. The contribution of this term to calculated values of $T_{1/2}$ turns out essential in some cases:

The spin and the parity of an α-particle are, respectively, $\pi_\alpha = +1$ and $I_\alpha = 0$. Therefore, by the selection rules for the α-decay by spin and parity, we have

$$|I_i - I_f| \leq \ell_\alpha \leq I_i + I_f; \quad \frac{\pi_i}{\pi_f} = (-1)^{\ell_\alpha}, \tag{2}$$

where (I_i, π_i) and (I_f, π_f) are, respectively, the spin and the parity of the initial and final states of the nucleus. In our calculations, we took, for simplicity, the least possible orbital moment of the transition, ℓ_α.

The α-decay half-life depends, in our approximation, on the charge Z and the mass number A of the parent nucleus, Q, the orbital moment of an α-particle ℓ_α, and the quantity $\mu = (A/(A-4))^{1/6}$ in the following way:

$$\log_{10}(T_{1/2}) = a_1 + a_2 \frac{A^{1/6} Z^{1/2}}{\mu} + a_3 \frac{Z}{\sqrt{Q}} +$$

$$+ a_4 \frac{\sqrt{\ell(\ell+1)}}{Q A^{-1/6}} + a_5((-1)^\ell - 1). \tag{3}$$

For even-even nuclei, $\ell_\alpha = 0$. Therefore, the coefficients a_4 and a_5 in formula (3) are equal to zero for e-e nuclei. The term $\sqrt{\ell(\ell+1)}$ describes the effect of the centrifugal potential, and the quantity $((-1)^\ell - 1)$ takes the complication of the α-emission with an odd value of ℓ into account. It is worth noting that the empirical approximations [3–9] do not consider these factors. Three first terms of the formula were determined from the following reasoning:

– first and third terms of function (3) appear as the derivatives of the Geiger–Nuttall relation which establishes the connection between the half-life of isotopes and the energy of an α-particle;

Table 1. Effect of the screening of a nucleus by electrons on the results of calculations of the α-decay half-lives. Columns 2-4 show the experimental values of the half-life, $T_{1/2}$, and those calculated with and without regard for the effect of screening, respectively

Isotope	$T^{\rm exp}_{1/2}$, s	$T_{1/2}$, s	$T_{1/2}$, s
$^{209}_{83}$Bi	5.996×10^{26}	8.673×10^{26}	2.249×10^{27}
$^{219}_{86}$Rn	4.987	7.532	10.29
$^{209}_{89}$Ac	0.092	0.088	0.118
$^{221}_{91}$Pa	5.90×10^{-6}	8.69×10^{-6}	$1.10\cdot10^{-5}$

– common logarithm of the half-life depends linearly on the term $A^{1/6}\sqrt{Z}/\mu$, which was revealed as a result of the search for various functional dependences $\log(\mathrm{T}_{1/2}) = f(A, Z, Q, \mu, \ell)$.

In Table 1, we give values of the parameters a_i $(i = 1, 5)$ in the empirical relation (3) for collections I-III. These values were obtained by the fitting of theoretical results to experimental values of the half-lives.

In Figure, we show the differences of experimental and theoretical values of common logarithms of the half-lives for even-even, even-odd, odd-even, and odd-odd nuclei for collection I of the parameters. We see that the differences of experimental and theoretical values of common logarithms of the half-lives are relatively small, especially for even-even nuclei.

The obtained results were compared with those of calculations by other empirical relations [3–9] for a single collection of data. The estimates of mean square errors of the α-decay half-lives for collections I-III,

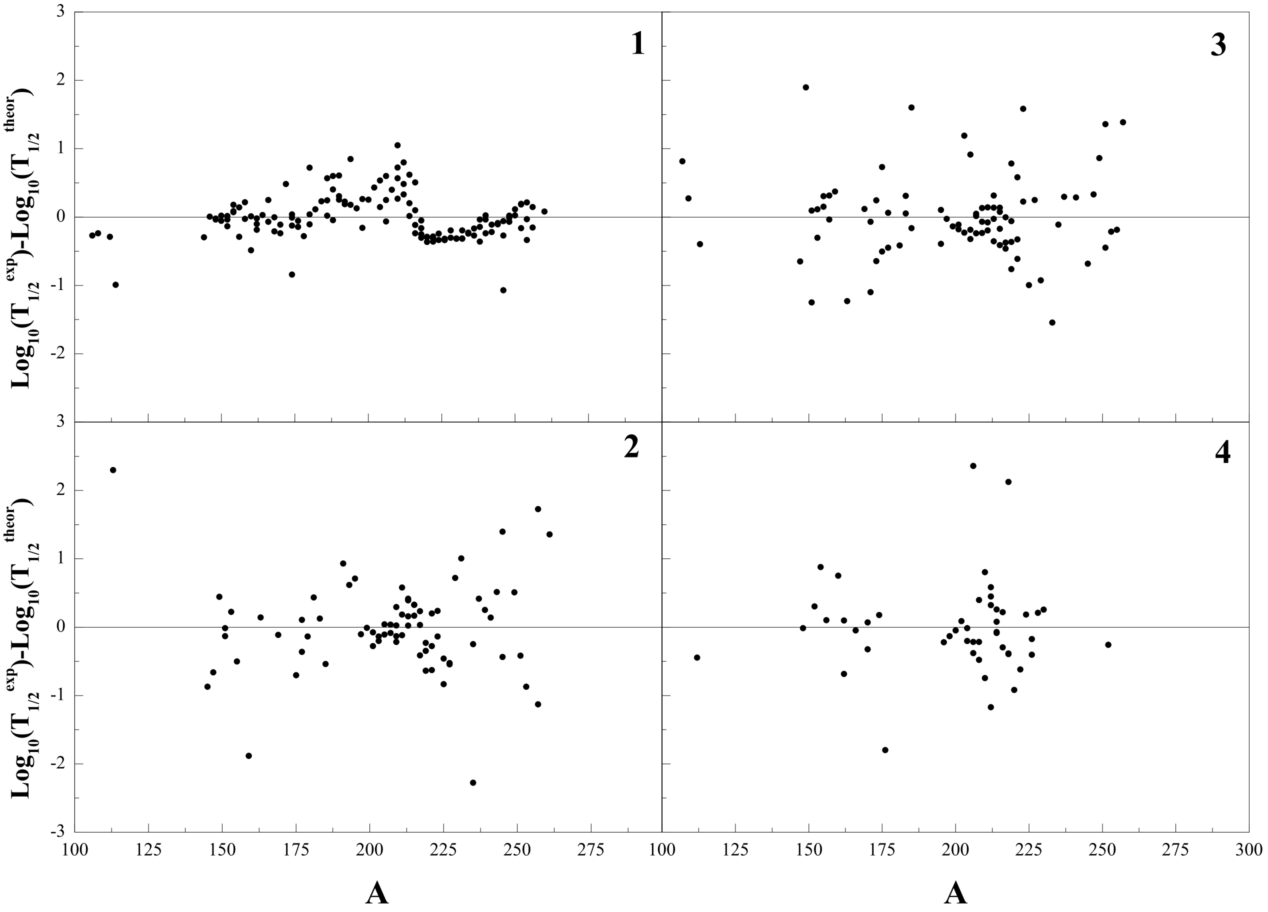
$$\delta = \sqrt{\frac{1}{N-1} \sum_{\mathrm{k}=1}^{\mathrm{N}} \left[\log_{10}(T^{\rm theor}_{1/2}) - \log_{10}(T^{\rm exp}_{1/2})\right]^2}, \tag{4}$$

are shown in Tables 2–4.

It follows from Tables 2-4 that, for the approximation presented in the given work, the errors are the least. We managed to improve the result by fitting the parameters of the function $\log_{10}(T_{1/2})$ for heavy and light nuclei separately. The errors for e-e nuclei are significantly less than those for e-o, o-e, and o-o ones, but this difference is significantly less for our results as compared

Table 2. Parameters a_i $(i = 1.5)$ in formula (3) for the full set of nuclei and for heavy and light nuclei

Nucleus	α_1	α_2	α_3	α_4	α_5
Full set of nuclei (Collection I)					
e-e	−26.1779	−1.1521	1.6068	0	0
e-o	−30.3391	−1.0785	1.6979	0.2688	−0.6784
o-e	−30.2138	−1.0841	1.6949	0.1302	−0.5972
o-o	−30.3526	−1.0149	1.6609	0.2762	−0.2209
Heavy nuclei(Collection II)					
e-e	−27.9238	−1.0521	1.5847	0	0
e-o	−34.9988	−0.8552	1.6822	0.2278	−0.6763
o-e	−33.5438	−0.9627	1.7077	0.1538	−0.5200
o-o	−38.8157	−0.5200	1.5645	0.5175	0.0287
Light nuclei (Collection III)					
e-e	−29.2230	−1.0347	1.6290	0	0
e-o	−29.3760	−1.0835	1.6711	0.3324	−6.2873
o-e	−28.7300	−1.1068	1.6652	0.1377	−0.6153
o-o	−31.5090	−1.0626	1.7298	0.1675	0.1080



Difference of the experimental and theoretical values $\log_{10}(T_{1/2}^{\text{exp}}) - \log_{10}(T_{1/2}^{\text{theor}})$ for even-even (*1*), even-odd (*2*), odd-even (*3*), and odd-odd (*4*) nuclei

with those for the other empirical approximations. We note that our purpose was the reliable determination of the third decimal point of mean square errors δ in Tables 2-4. Therefore, we used 4 decimal points for the parameters a_i (see Table 1).

It is worth noting that the original works [3–9] used other collections of data and other nuclei. In addition, we note that the data are permanently renewed. Therefore, different databases for the same nucleus have different characteristics. In our calculations, we considered the transitions only between the ground states of par-

Table 3. Mean square errors of the α-decay half-lives: the total one and those for e-e, e-o, o-e and o-o, nuclei from collection I

Total	e-e	e-o	o-e	o-o	
0.5484	0.3314	0.6237	0.6768	0.6792	Collection I
0.6248	0.3088	0.7816	0.7621	0.7546	[11]
1.0113	0.4164	1.3548	1.2572	1.0965	[9]
1.0185	0.5165	1.1611	1.3348	1.2568	[5]
1.1130	0.3837	1.4762	1.3688	1.3340	[6]
1.1285	0.3712	1.5425	1.3541	1.3307	[3]
1.3813	1.2928	1.4300	1.5607	1.2751	[4]

Table 4. Mean square errors of the α-decay half-lives: the total one and those for e-e, e-o, o-e, and o-o nuclei from collection II

Total	e-e	e-o	o-e	o-o	
0.5369	0.1905	0.6739	0.7632	0.5620	Collection II
0.5702	0.2677	0.6937	0.7757	0.6457	Collection I
0.7170	0.3135	0.9520	0.9184	0.8032	[11]
1.2326	0.2854	1.8008	1.4748	1.4753	[4]
1.2516	0.3861	1.6558	1.5062	1.7615	[5]
1.2543	0.2686	1.9013	1.5686	1.1856	[9]
1.3410	0.3067	2.0223	1.6186	1.4219	[7]
1.4399	0.2202	2.1371	1.6545	1.8339	[6]
1.4933	0.3701	2.2528	1.6663	1.8292	[3]
1.6926	0.2187	2.5050	1.9202	2.2285	[8]

Table 5. Mean square errors of the α-decay half-lives: the total one and those for e-e, e-o, o-e, and o-o nuclei from collection III

Total	e-e	e-o	o-e	o-o	
0.4960	0.2692	0.5733	0.5869	0.6667	Collection III
0.5336	0.3747	0.5811	0.5947	0.7094	Collection I
0.5509	0.3071	0.6588	0.6192	0.7381	[11]
0.7699	0.3744	0.8375	1.0579	0.9532	[3]
0.7817	0.4463	0.8563	0.9544	1.0580	[9]
0.8034	0.4738	0.8334	1.1064	0.9552	[6]
0.8138	0.6001	0.6952	1.1971	0.8607	[5]
1.4822	1.7049	1.1484	1.6447	1.1659	[4]

ent and daughter nuclei, as distinct from some above-mentioned works, and took the influence of the orbital moment of the α-transition and the effect of screening of nuclei by electrons into account. These factors affect the results of calculations and causes a significant difference of the results of the given approximation and the relations of the original works [3–9].

4. Conclusions

In order to determine a simple empirical dependence for the α-decay half-life, we have used experimental data for 344 nuclei. The analysis was carried out for the full collection of isotopes and separately for heavy/light nuclei. The comparison of mean square errors for our calculations with errors for other empirical approximations for our collection of data indicates a significant improvement of the results in the frame of our empirical functions as compared with those of the previous works.

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ЕМПІРИЧНІ СПІВВІДНОШЕННЯ ДЛЯ ПЕРІОДІВ АЛЬФА-РОЗПАДУ

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Резюме

Знайдено емпіричні співвідношення періодів альфа-переходів між основними станами материнського та дочірнього ядер з парною/непарною кількістю протонів/нейтронів. Проаналізовано експериментальні дані 344 альфа-радіоактивних ізотопів. Знайдено відповідні залежності для повного набору ядер, окремо для важких (важчих за ^{208}Pb) та легких ядер. Враховано ефект екранування ядра електронами та наявність відмінного від нуля орбітального моменту альфа-частинки для розпаду парно-непарних, непарно-парних та непарно-непарних ізотопів.