ISSN 1062-8738, Bulletin of the Russian Academy of Sciences: Physics, 2010, Vol. 74, No. 6, pp. 782–785. © Allerton Press, Inc., 2010.
Original Russian Text © V.Yu. Denisov, N.A. Pilipenko, 2010, published in Izvestiya Rossiiskoi Akademii Nauk. Seriya Fizicheskaya, 2010, Vol. 74, No. 6, pp. 818–822.

The Interaction and Fusion of Arbitrarily Oriented Deformed Nuclei

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Abstract—The fusion/capture cross sections between various deformed nuclei are calculated. It is shown that quadrupole and hexadecapole deformations of heavy nuclei are important for evaluating the barrier height, capture well depth, and fusion/capture cross sections of heavy nuclei. It is found that calculations of the capture cross section of two heavy deformed nuclei must be performed with averaging over all possible mutual orientations of the colliding nuclei.

DOI: 10.3103/S1062873810060110

INTRODUCTION

Deformed nuclei are involved in various nuclear reactions [1, 2]. Detailed studies of subbarrier fusion reactions have revealed that their cross sections exceed considerably the predictions of the one-dimensional tunneling model [1–10]. Various models and mechanisms of the fusion reaction have been proposed to describe the substantial increase in cross section at subbarrier energies [1–10]. It is therefore of great interest to study carefully the nature of the reactions between deformed nuclei at close-to-barrier energies.

The orientation of deformed nuclei has an appreciable effect on the barrier height [3, 6, 7, 11, 12] and the shape of a nucleus–nucleus potential well [11, 12]. For example, in reactions between the spherical ^{48}Ca nucleus and the deformed heavy ^{238}U, ^{244}Pu, and ^{248}Cm nuclei that are used to form superheavy nuclei [13], the change in barrier height at different orientations of the deformed nucleus exceeds 20 MeV [11]. The changes in barrier height at different orientations of two deformed heavy nuclei are even larger [12].

The analytical relations for calculating the Coulomb and nuclear interaction energies for arbitrarily oriented deformed nuclei were obtained in [12]. These relations were derived by taking into account the quadratic terms in the nuclear quadrupole deformation β_2^2 and the linear terms in higher deformations: β_L, $L > 2$. In calculating the fusion of two deformed nuclei, it is necessary to average the cross section over all possible mutual orientations of colliding nuclei, since collisions can occur at different orientations under experimental conditions. It would therefore be useful to calculate the fusion cross sections of two deformed nuclei as the functions of near-barrier deformation using the nucleus–nucleus potential from [12] and averaging over all possible orientations of the colliding nuclei. Note that in [3, 6, 7] the nucleus–nucleus potential was calculated ignoring certain terms that are quadratic in nuclear deformation, and the calculated cross sections were averaged only over certain specific orientations of the colliding nuclei.

1. THE FUSION OF DEFORMED NUCLEI

Colliding nuclei can have various mutual orientations; it is therefore necessary to average the reaction cross section over all possible mutual orientations of the deformed nuclei:

$$\sigma(E) = \frac{\pi\hbar^2}{2\mu E}\sum_l (2l+1)\langle T_l(E,\theta_1,\theta_2,\varphi)\rangle.$$

Here, μ is the reduced mass, E is the collision energy, and $\langle T_l(E,\theta_1,\theta_2,\varphi)\rangle$ is the transmission coefficient for the lth partial wave, averaged over all possible angles $\theta_1, \theta_2,$ and φ, which determine the mutual orientation of deformed nuclei in space [12].

At subbarrier energies, and for certain nuclear orientations, the transmission coefficient is calculated within the Wentzel–Kramers–Brillouin approximation:

$$T_l(E,\theta_1,\theta_2,\varphi) = \left\{1+\exp\left[\frac{2}{\hbar}\int_a^b dR\sqrt{2\mu\left(V_C(R,\theta_1,\theta_2,\varphi)+\eta V_N(R,\theta_1,\theta_2,\varphi)+\frac{\hbar^2 l(l+1)}{2\mu R^2}-E\right)}\right]\right\}^{-1},$$

where a and b are the turning points; $V_C(R,\theta_1,\theta_2,\varphi)$ and $V_N(R,\theta_1,\theta_2,\varphi)$ are, respectively, the Coulomb and

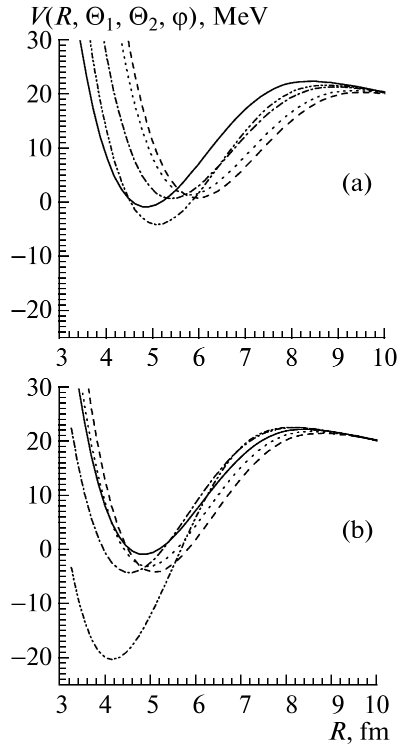

Fig. 1. Interaction potentials for different mutual orientations of ^{24}Mg + ^{24}Mg nuclei. The orientation of one nucleus is fixed at either (a) $\theta_1 = 0°$ or (b) $\theta_1 = 90°$, while the other nucleus has the orientation $\theta_2 = 0°$, 30°, 60°, or 90° (the corresponding calculation results are shown by dashed, dotted, dotted-and-dashed, and double-dotted-and-dashed lines, respectively). The interaction potential for spherical ^{24}Mg + ^{24}Mg nuclei ($\beta_1 = \beta_2 = 0$) is shown by solid lines for the sake of comparison.

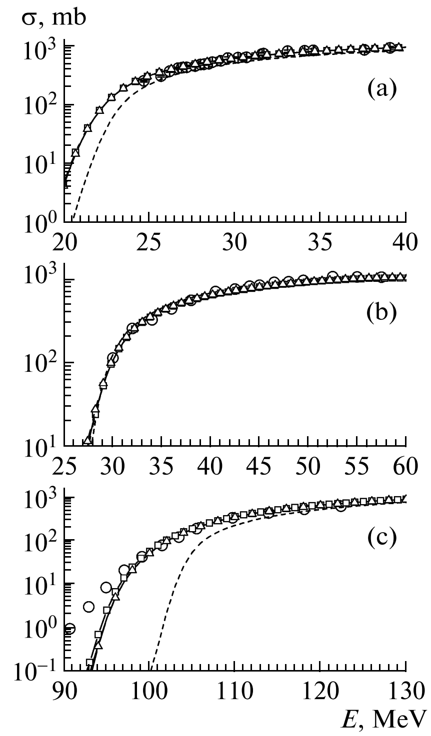

Fig. 2. Fusion cross sections for the (a) ^{24}Mg + ^{24}Mg (a), ^{28}Si + ^{28}Si, and (c) ^{154}Sm + ^{28}Si reactions near the barrier. Circles represent the experimental data; solid lines, calculations involving the linear and quadratic terms in the quadrupole deformation of both nuclei; dashed lines, calculations within the spherical-nucleus approximation; solid lines with squares, calculations involving only the linear terms in the quadrupole deformation of both nuclei; and solid lines with triangles, calculations disregarding averaging over the angle φ at the fixed value $\varphi = 0°$.

nuclear parts of the interaction potential of the nuclei [12]; and η is the fitting coefficient that determines the relative strength of the nuclear part of the total potential. The coefficient η is close to unity; it is chosen so as to satisfactorily describe the above-barrier fusion, in which the deformation and fine mechanisms of the nuclear reaction are not important. Note that the radicand is zero at the turning points a and b. At above-barrier energies, the transmission coefficient was determined in the Hill–Wheeler approximation [14].

Figure 1 shows the ^{24}Mg + ^{24}Mg interaction potentials for different nuclear orientations. The ^{24}Mg nucleus is elongated ($\beta_2 = 0.438$). Here and below, we use the nuclear quadrupole deformations from [15]. One nucleus has a fixed orientation ($\theta_1 = 0°$ or 90°), while the other has the orientation $\theta_2 = 0°$, 30°, 60°, or 90°. Note that for the orientations $\theta_1 = \theta_2 = 0°$ and $\theta_1 = \theta_2 = 90°$, the barrier height is at its minimum and maximum, respectively. The barrier height calculated for the spherical ^{24}Mg + ^{24}Mg nuclei is intermediate between these two limiting cases. Figure 2 shows the ^{24}Mg + ^{24}Mg fusion cross sections obtained for different calculated nucleus–nucleus potentials. The cross sections determined in the spherical nucleus approximation are much smaller than the experimental data in [16] at low collision energies.

Note that the ^{24}Mg nucleus is elongated. To determine the effect of the deformation type (elongated or flattened) on the increase in cross section at subbarrier energies, we compared (see Fig. 2) the fusion cross sections calculated within different approximations using the experimental data [17, 18] for the ^{28}Si + ^{28}Si and ^{154}Sm + ^{28}Si systems. The ^{154}Sm nucleus is elongated ($\beta_2 = 0.34$), while the ^{28}Si nucleus is flattened ($\beta_2 = -0.407$). A comparison of the calculation results for the ^{24}Mg + ^{24}Mg, ^{28}Si + ^{28}Si, and ^{154}Sm + ^{28}Si systems, obtained with allowance for deformation and with the results found in the spherical-nucleus approximation, suggests that a flattened deformation in light nuclei leads to an inconsiderable increase in the fusion cross section at low energies, whereas an elongated deformation significantly increases the

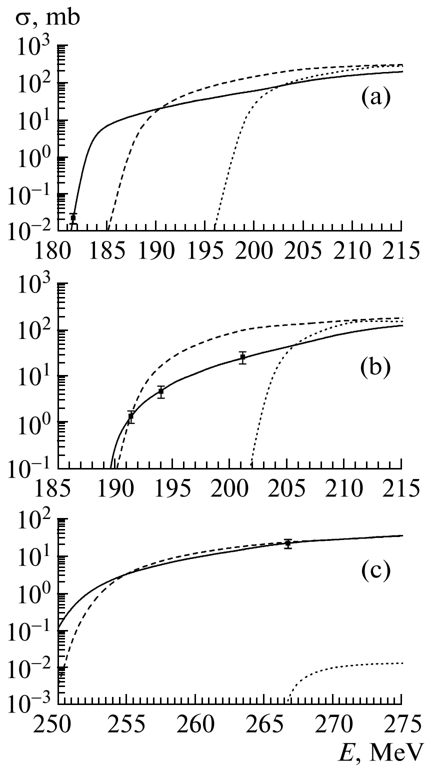

Fig. 3. Capture cross sections of the (a) $^{48}Ca + {}^{238}U$, (b) $^{48}Ca + {}^{244}Pu$, and (c) $^{58}Fe + {}^{248}Cm$ reactions near the barrier. Barred squares represent the experimental data; solid lines, calculations involving the linear and quadratic terms in quadrupole and hexadecapole nuclear deformations; dashed lines, calculations involving the linear and quadratic terms in quadrupole deformation and disregarding hexadecapole deformation; and dotted lines, calculations within the spherical-nucleus approximation.

cross section at subbarrier energies. The increase grows larger with an increase in the mass number.

The effect of the second-order terms in the quadrupole deformation (β_2^2) on the nuclear fusion cross section is indistinguishable for the light systems shown in Fig. 2 and almost nonexistent for the $^{154}Sm + {}^{28}Si$ system. For all of the systems considered in this section, the effect of averaging over the orientational angle φ is negligible.

2. THE CAPTURE OF HEAVY DEFORMED NUCLEI

The capture cross sections for heavy nuclear systems are of particular interest, since superheavy elements are synthesized in these reactions, and the nuclear capture related to the formation of a system of coalescing contacting nuclei is the initial stage of this synthesis [11, 12, 19, 20]. The capture cross sections of heavy nuclei are calculated in a manner similar to the one used for the fusion cross sections in the previous section. These cross sections coincide for light systems because the contribution of quasi-fission is negligible; however, these cross sections differ significantly for heavy nuclei. The experimental capture cross sections were measured in [20] for heavy asymmetric nuclear systems. We will therefore first consider the capture cross sections for the $^{48}Ca + {}^{238}U$, $^{48}Ca + {}^{244}Pu$, and $^{58}Fe + {}^{248}Cm$ reactions. It is also of interest to study the capture cross sections for heavy nuclei with similar masses. A characteristic example of such a reaction is given at the end of this subsection.

In Fig. 3, the capture cross sections for the $^{48}Ca + {}^{238}U$, $^{48}Ca + {}^{244}Pu$, and $^{58}Fe + {}^{248}Cm$ reactions are compared with the experimental data [20]. The cross sections calculated using the second-order terms in quadrupole deformation and the values of hexadecapole nuclear deformation are in satisfactory agreement with the experiments. The values of hexadecapole deformation were taken from [21]. The hexadecapole deformation is small, but it has an appreciable effect on the capture cross sections of heavy nuclei at low energies, since the hexadecapole deformation increases the depth of capture well at certain nuclear orientations. Note that the capture-well depth in such heavy systems is shallow, while for light systems the nuclear capture depths are much greater [11, 12].

The spherical approximation for colliding nuclei yields substantially underestimated capture cross sections. The reason is that the potential barrier height calculated for spherical nuclei considerably exceeds the barrier height determined with allowance for the deformation of heavy nucleus [11, 12]. In addition, the depth of a potential well calculated in the spherical-nucleus approximation is less than the depth found when nuclear deformation is taken into account [11, 12]. As a result, the spherical potential well can "capture" fewer partial waves, leading to a decrease in the cross section. This effect is especially pronounced in the $^{58}Fe + {}^{248}Cm$ collision. In the case of a collision of nuclei with similar masses leading to the formation of very heavy nuclear systems, the well depth and capture cross section are reduced. Figure 4 shows the capture cross section for two deformed nuclei, $^{150}Nd + {}^{158}Gd$. In this situation, consideration of the hexadecapole deformation and averaging over all angles describing the mutual orientation of the nuclei are important in calculating the potential and fusion cross section. For example, consideration of the hexadecapole deformation leads to a considerable increase in well depth at certain mutual orientations of heavy deformed nuclei (see [12]) and, therefore, to an increase in the number of captured partial waves. As a result, the capture cross section is substantially increased.

The barrier height and well depth in heavy systems depend on the angle φ. Choosing the nuclear orientation related to a certain φ value and disregarding averaging over the angle φ, we can calculate the capture cross sections, which turn out to be different (Fig. 4).

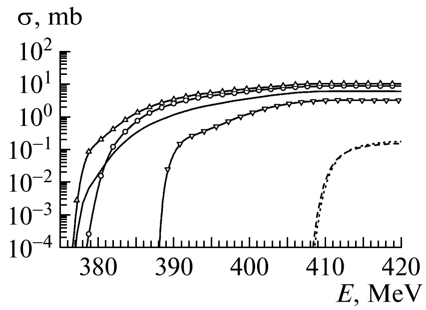

Fig. 4. Capture cross section of the ^{150}Nd + ^{158}Gd reaction near the barrier. The solid line represents calculations involving all linear and quadratic terms in quadrupole deformation, the linear terms in hexadecapole nuclear deformation, and averaging over the angles θ_1, θ_2, and φ; the dashed line, calculations involving quadrupole deformation and disregarding hexadecapole deformation; and the dotted line, calculations involving only the linear terms in quadrupole deformation. The solid lines with upright triangles, inverted triangles, and circles represent calculations disregarding averaging over the angle φ at the fixed values $\varphi = 0°$, $90°$, and $180°$, respectively.

If we ignore the hexadecapole nuclear deformation, consideration of the quadratic terms in the quadrupole deformation in the nucleus–nucleus potential leads to inappreciable variations in the potential, in comparison with calculations that include only the linear terms in the quadrupole deformation. The capture cross sections therefore also change only slightly in these cases. The role of these terms increases, however, when the hexadecapole nuclear deformation is considered (see, e.g., the dependence of the cross section on the angle φ, which is associated with terms proportional to the product of the quadrupole deformations of the colliding nuclei).

Note that the capture cross section for the ^{150}Nd + ^{158}Gd reaction in the spherical-nucleus approximation is zero because the capture well in the nuclear interaction potential is absent in this case.

CONCLUSIONS

(i) The deformation of nuclei and their relative orientation substantially affect the nucleus–nucleus interaction potential, specifically the barrier height and position and the width and depth of a capture well.

(ii) The fusion cross section of deformed nuclei depends strongly on the nuclear deformation, especially at subbarrier energies.

(iii) Consideration of the quadratic terms in the quadrupole deformation (β_2^2) and the linear terms in the hexadecapole deformation (β_4) in the nucleus–nucleus potential is very important for describing the capture of heavy nuclei.

(iv) The influence of the deformation and orientational effects on the subbarrier fusion cross section increases with an increase in the nuclear charge and mass.

REFERENCES

1. Bass, R., *Nuclear Reactions with Heavy Ions,* Berlin: Springer–Verlag, 1980.
2. Frobrich, P. and Lipperheide, R., *Theory of Nuclear Reactions*, Oxford: Clarendon, 1996.
3. Wong, C.Y., *Phys. Rev. Lett.*, 1973, vol. 31, p. 766.
4. Beckerman, M., *Phys. Rep.*, 1985, vol. 129, p. 145; *Rep. Prog. Phys.*, 1988, vol. 51, p. 1047.
5. Dasgupta, M., et al., *Annu. Rev. Nucl. Part. Sci.*, 1998, vol. 48, p. 401.
6. Fernandez-Niello, J.O., Dasso, C.H., and Landowne, S., *Comput. Phys. Commun.*, 1989, vol. 54, p. 409.
7. Dasso, C.H., Fernandez-Niello, J.O., and Landowne, S., *Phys. Rev. C,* 1990, vol. 41, p. 1014.
8. Denisov, V.Yu., *Phys. Rev. C*, 2006, vol. 74, p. 055804.
9. Denisov, V.Yu., *Yad. Fiz.*, 1991, vol. 54, p. 1556; Denisov, V.Yu. and Rouer, G., *J. Rhus. G*, 1994, vol. 20, p. L43; Denisov, V.Yu. and Reshit'ko, S.V., *Yad. Fiz.,* 1996, vol. 59, p. 78 [*Phys. At. Nucl.* (Engl. Transl.), 1996, vol. 59, no. 1, p. 72].
10. Denisov, V.Yu., *Eur. Phys. J. A*, 2000, vol. 7, p. 87.
11. Denisov, V.Yu. and Norenberg, W., *Eur. Phys. J. A*, 2002, vol. 15, p. 375.
12. Denisov, V.Yu. and Pilipenko, N.A., *Phys. Rev. C,* 2007, vol. 76, p. 014602.
13. Oganessian, Yu., *J. Geom. Phys.*, 2007, vol. 34, p. R165.
14. Hill, D.L. and Wheeler, J.A., *Phys. Rev.,* 1953, vol. 89, p. 1102.
15. http://cdfe.sinp.msu.ru
16. Jachcinski, C.M., et al., *Phys. Rev. C*, 1981, vol. 24, p. 24.
17. Gil, S., et al., *Phys. Rev. Lett.,* 1990, vol. 65, p. 3110.
18. Nagashima, Y., et al., *Phys. Rev. C*, 1986, vol. 33, p. 33.
19. Denisov, V.Yu. and Hofmann, S., *Phys. Rev. C,* 2000, vol. 61, p. 034606.
20. Itkis, M.G., et al., *Nucl. Phys. A*, 2007, vol. 787, p. 150c.
21. Moller, P., et al., *Atomic Data Nucl. Data Tables*, 1995, vol. 59, p. 195.