Fusion of deformed nuclei: $^{12}C + ^{12}C$

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The S factor and the fusion cross-section values for the $^{12}C + ^{12}C$ reaction are evaluated in the framework of a simple barrier-penetration model, which takes into account quadrupole and hexadecapole surface deformations of ^{12}C. The calculated values agree well with corresponding experimental data at off-resonances energies. It is shown that the strong oblate quadrupole deformation of ^{12}C induces the increasing of both the S factor and the fusion cross-section values at subbarrier energies. The S factor and the fusion cross section for the $^{12}C + ^{12}C$ reaction are slightly reduced at low energy due to the influence of the hexadecapole deformation.

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I. INTRODUCTION

The fusion reaction $^{12}C + ^{12}C$ is very important for astrophysics because this reaction is related to carbon burning in stars, nucleosynthesis of ^{20}Ne and ^{23}Na, carbon flashes on accreting neutron stars, pycnonuclear reaction in white dwarfs, and so on. [1]. Due to its astrophysical importance this reaction has been measured many times around and well-below barrier by various experimental groups over the past 40 years [2–12]. The cross section of fusion reaction $^{12}C + ^{12}C$ was extensively discussed in the framework of various approximations, see Refs. [3,4,8,10,13–17] and articles cited therein.

It is well known that the ground-state shape deformation of nuclei plays an important role for subbarrier fusion reactions [18–22] and various capture reactions in stars [23] because the interaction potential between nuclei with deformed ground states depends on the orientation of incoming nuclei [18,19,21,22,24]. Various barrier heights take place at various mutual orientations of colliding deformed nuclei. Small barrier heights lead to high subbarrier transmission and as a result to the enhancement of the fusion cross section at very low energies. As a result of this the fusion cross section for heavy systems strongly depends on the shape deformation at very low collision energies. Note that a recent study of the $^{12}C + ^{12}C$ molecular shell structure in the framework of the two-center shell model with arbitrarily oriented deformed nuclei [25] shows that nonaxial symmetric configurations play a crucial role in molecular resonances observed for the $^{12}C + ^{12}C$ reaction [2–12,26].

The ground-state shape of ^{12}C is well deformed. The values of the quadrupole and hexadecapole deformation parameters are, respectively, $\beta_2 = -0.40 \pm 0.02$ and $\beta_4 = 0.16 \pm 0.03$ [27]. However, the shape of the ^{12}C ground state is considered to be spherical in previous studies of the fusion cross section or the S factor for the $^{12}C + ^{12}C$ reaction. Therefore, it is very important to evaluate the $^{12}C + ^{12}C$ fusion cross section in the framework of theory, which takes into account the surface deformation exactly because very precise values of the S factor at low energies are needed for the evaluation of various astrophysical phenomena.

We evaluate the fusion cross section of the $^{12}C + ^{12}C$ reaction in the framework of a simple barrier-penetration approach, which takes into account the quadrupole and hexadecapole deformations of the ^{12}C ground state and the various mutual orientations of colliding deformed nuclei that occurred during the fusion reaction. Unfortunately, we cannot describe the molecular resonances observed for the $^{12}C + ^{12}C$ reaction [2–12,26] in the framework of a simple barrier-penetration model. Therefore, our aim is to discuss an overall energy dependence of the S factor or fusion cross section related to off-resonances energies and evaluate the effect induced by deformation of the ^{12}C ground state on both the S factor and the fusion cross section of the $^{12}C + ^{12}C$ reaction.

It is well known that the couplings to the surface vibrational states strongly increase the fusion cross sections between heavy ions at subbarrier energies [20]. However, no effect of the couplings is observed for the fusion cross section for the $^{12}C + ^{12}C$ reaction [15] because the coupling strength for the $^{12}C + ^{12}C$ system is small and the excitation energies of 2^+ and 3^- states are large, respectively, 4.44 and 9.64 MeV [28]. Therefore, we did not take into account the couplings to the surface vibrational states at the evaluation of the fusion cross section for the $^{12}C + ^{12}C$ reaction.

We briefly discuss our approach for the evaluation of the fusion cross section in Sec. II. A discussion and conclusion are given in Sec. III.

II. FUSION OF DEFORMED NUCLEI

Various orientations of deformed nuclei occur during collisions; therefore the fusion reaction cross section induced by two deformed nuclei should be averaged over all possible orientations of colliding nuclei

$$\sigma(E) = \frac{\pi\hbar^2}{2\mu E}\sum_{\ell}(2\ell+1)\frac{1}{8\pi}\int_0^{\pi}\sin(\Theta_1)d\Theta_1 \times \int_0^{\pi}\sin(\Theta_2)d\Theta_2\int_0^{2\pi}d\Phi\, T(E,\ell,\Theta_1,\Theta_2,\Phi). \quad (1)$$

Here μ is the reduced mass of the colliding nuclei, E is the collision energy, and $T(E, \ell, \Theta_1, \Theta_2, \Phi)$ is the transmission coefficient evaluated at the orientation of the colliding nuclei specified by angles Θ_1, Θ_2, and Φ (see Fig. 1).

We use the Wentzel-Kramers-Brillouin (WKB) approximation for the evaluation of the transmission coefficient for subbarrier energies

$$T(E, \ell, \Theta_1, \Theta_2, \Phi) = \left\{1 + \exp\left[\frac{2}{\hbar}\int_{a(E,\ell,\Theta_1,\Theta_2,\Phi)}^{b(E,\ell,\Theta_1,\Theta_2,\Phi)} \sqrt{2\mu[V(R, \ell, \Theta_1, \Theta_2, \Phi) - E]}\, dR\right]\right\}^{-1}, \tag{2}$$

and the Hill-Wheeler approach [29] for over-barrier collision energies. The inner $a(E, \ell, \Theta_1, \Theta_2, \Phi)$ and outer $b(E, \ell, \Theta_1, \Theta_2, \Phi)$ turning points in Eq. (2) are determined from corresponding equations

$$V[a(E, \ell, \Theta_1, \Theta_2, \Phi), \ell, \Theta_1, \Theta_2, \Phi] = E, \tag{3}$$

$$V[b(E, \ell, \Theta_1, \Theta_2, \Phi), \ell, \Theta_1, \Theta_2, \Phi] = E. \tag{4}$$

The interaction potential $V(R, \ell, \Theta_1, \Theta_2, \Phi)$ of two deformed nuclei at distance R between mass centers and the mutual orientation described by angles Θ_1, Θ_2, and Φ consists of Coulomb $V_{\rm C}(R, \Theta_1, \Theta_2, \Phi)$, nuclear $V_{\rm N}(R, \Theta_1, \Theta_2, \Phi)$, and rotational $V_\ell(R) = \hbar^2\ell(\ell+1)/(2\mu R^2)$ parts

$$V(R, \ell, \Theta_1, \Theta_2, \Phi) = V_{\rm C}(R, \Theta_1, \Theta_2, \Phi) + V_{\rm N}(R, \Theta_1, \Theta_2, \Phi) + V_\ell(R). \tag{5}$$

The Coulomb interaction of two deformed nuclei is approximated as [24]

$$\begin{aligned} V_{\rm C}(R, \Theta_1, \Theta_2, \Phi) &= \frac{Z_1 Z_2 e^2}{R}\Bigg\{1 + \sum_{\ell\geqslant 2}[f_{1\ell}(R, \Theta_1, R_{10})\beta_{1\ell} \\ &\quad + f_{1\ell}(R, \Theta_2, R_{20})\beta_{2\ell}] + f_2(R, \Theta_1, R_{10})\beta_{12}^2 \\ &\quad + f_2(R, \Theta_2, R_{20})\beta_{22}^2 + f_3(R, \Theta_1, \Theta_2, R_{10}, R_{20})\beta_{12}\beta_{22} \\ &\quad + f_4(R, \Theta_1, \Theta_2, \Phi, R_{10}, R_{20})\beta_{12}\beta_{22}\Bigg\}, \end{aligned} \tag{6}$$

where Z_1 and Z_2 are the number of protons in corresponding nuclei, $\beta_{i\ell}$ is the parameter of the ℓ-pole deformation of nucleus i ($i = 1, 2$), and $f_{1\ell}(R, \Theta_i, R_{i0})$, $f_2(R, \Theta_i, R_{i0})$, $f_3(R, \Theta_1, \Theta_2, R_{10}, R_{20})$, and $f_4(R, \Theta_1, \Theta_2, \Phi, R_{10}, R_{20})$ are simple functions [24]. Here R_{i0} is the radius of nucleus i in the case of spherical form. Note that we take into account the volume correction appearing in the second order on the quadrupole deformation parameter. The values of the deformation parameters of nuclei at the ground state satisfy the condition $\beta_{i2}^2 \approx \beta_{i\ell}|_{\ell\geqslant 3}$ as a rule; therefore the lowest correction terms for expression (6) are proportional to $\mathcal{O}(\beta_{i2}^3)$ or $\mathcal{O}(\beta_{i2}\beta_{i\ell})|_{\ell\geqslant 3}$.

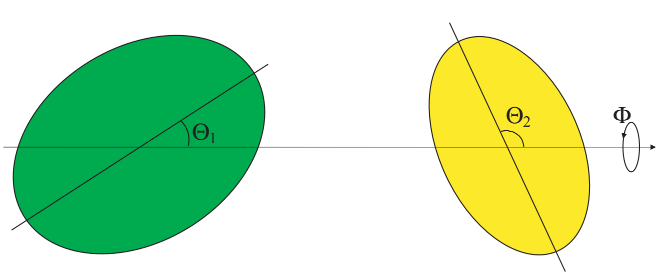

FIG. 1. (Color online) Angles Θ_1, Θ_2, and Φ, which determine the mutual orientation of two axial-symmetric nuclei in the space.

Applying the proximity theorem [30] we can obtain a simple parametrization of the nuclear part of the interaction potential between two deformed nuclei [24]

$$V_{\rm N}(R, \Theta_1, \Theta_2, \Phi) \approx \frac{1/R_{10} + 1/R_{20}}{[(C_1^{\parallel} + C_2^{\parallel})(C_1^{\perp} + C_2^{\perp})]^{1/2}} \times V_{\rm N}^{\rm sph}[d(R, \Theta_1, \Theta_2, \Phi, \beta_{i2}, \beta_{i\ell})], \tag{7}$$

where $C_i^{\parallel}$ and $C_i^{\perp}$ are the main curvatures of the deformed surface of nucleus i at the point closest to the surface of another nucleus, $d(R, \Theta_1, \Theta_2, \Phi, \beta_{i2}, \beta_{i\ell})$ is the closest distance between surfaces of interacting nuclei, and $V_{\rm N}^{\rm sph}(d)$ is the nuclear part of the interaction potential between spherical nuclei at $d = R - R_{10} - R_{20}$. The nuclear part of the potential depends strongly on the value of the closest distance between surfaces of interacting nuclei; therefore we evaluate $d(R, \Theta_1, \Theta_2, \Phi, \beta_{i2}, \beta_{i\ell})$ numerically.

The nuclear part of the interaction potential between spherical nuclei $V_{\rm N}^{\rm sph}(d)$ is described by Eqs. (10)–(14) in Ref. [31]. This expression for the nuclear part of the potential is obtained using the semi-microscopic energy density approach for the evaluation of nucleus-nucleus interaction energy [31,32]. The barrier height and barrier radius of the potential between spherical nuclei evaluated with the help of this expression agree well with corresponding empirical values [31].

The surface curvatures $C_i^{\parallel(\perp)}$ depend on corresponding orientation angle(s) and deformation parameters. Useful expressions for surface curvatures are given in Ref. [24]. Note that effects of surface deformations on the nuclear part of the interaction between nuclei are considered with the same accuracy as the one for the Coulomb part.

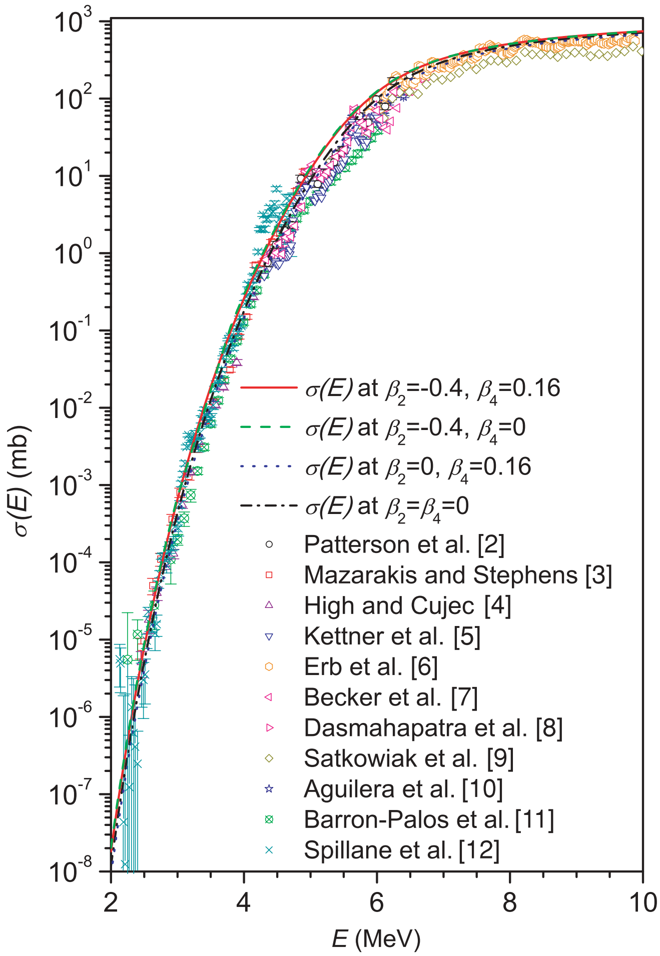

FIG. 2. (Color online) $^{12}C + ^{12}C$ fusion cross section evaluated for various approximations on the surface shape of the ^{12}C ground state are compared with the experimental data taken from Patterson *et al.* [2], Mazarakis and Stephens [3], High and Cujec [4], Kettner *et al.* [5], Erb *et al.* [6], Becker *et al.* [7], Dasmahapatra *et al.* [8], Satkowiak *et al.* [9], Aguilera *et al.* [10], Barron-Palos *et al.* [11], and Spillane *et al.* [12].

III. DISCUSSION AND CONCLUSION

Using Eqs. (1) through (7), we evaluate the fusion cross-section $\sigma(E)$ values for the $^{12}C + ^{12}C$ reaction. The S factor is proportional to the cross section of the reaction [1,10,13,23]

$$S(E) = E\ \exp(2\pi\eta(E))\,\sigma(E), \tag{8}$$

where $\eta(E) = Z_1 Z_2 e^2/(\hbar v)$ is the Sommerfeld parameter and $v = (2E/\mu)^{1/2}$ is the relative velocity of the colliding nuclei at the infinite distances between them.

We compare the fusion cross-section and S-factor values for the $^{12}C + ^{12}C$ reaction evaluated in the framework of our model with experimental data from Refs. [2–12] in Figs. 2 and 3. Unfortunately the values of the fusion cross section measured by various experimental groups are different. Nevertheless, the results of our calculations are in good agreement with most of the experimental data for the off-resonances energies in the full energy range where the cross-section values vary over ten orders. Note that we cannot make any fitting procedure and thus use the same set of parameters for the nuclear part of the interaction between nuclei as in Ref. [31].

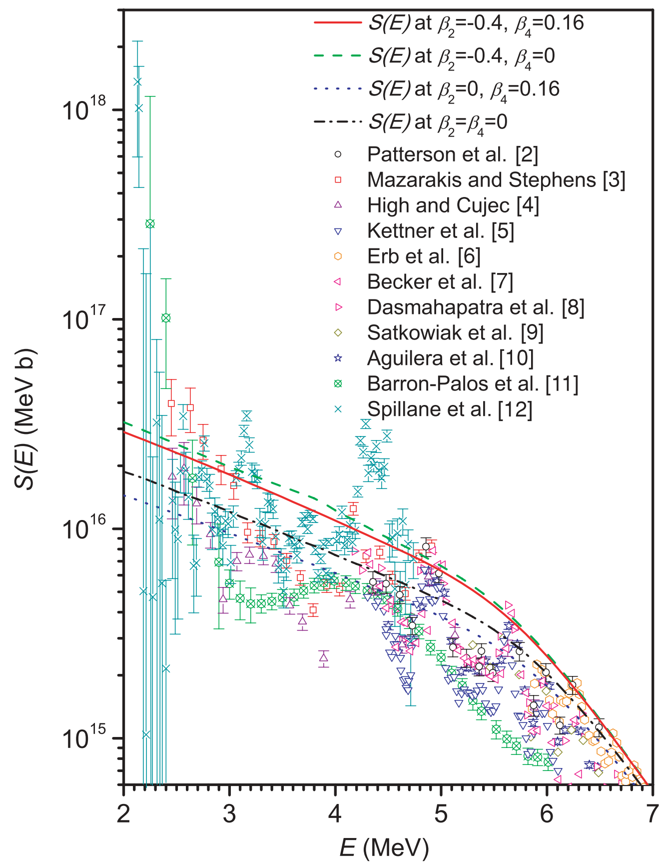

FIG. 3. (Color online) S factor for the $^{12}C + ^{12}C$ reaction evaluated for various approximations on the surface shape of the ^{12}C ground state are compared with the experimental data taken from Patterson *et al.* [2], Mazarakis and Stephens [3], High and Cujec [4], Kettner *et al.* [5], Erb *et al.* [6], Becker *et al.* [7], Dasmahapatra *et al.* [8], Satkowiak *et al.* [9], Aguilera *et al.* [10], Barron-Palos *et al.* [11], and Spillane *et al.* [12].

For clarification of the deformation effect and the role of the quadrupole and/or hexadecapole deformations on the fusion cross-section and S-factor values, we present the results obtained by using various approximations for the ground-state shape of the ^{12}C surface in Figs. 2 and 3. The results obtained by applying various approximations for the ^{12}C ground-state surface shape are very close to each other at high (over-barrier) collision energies $E \gtrsim 6$ MeV. However, the deformation effects are clearly observed for the S-factor values at subbarrier collision energies (see Fig. 3). The cross-section values vary over ten orders in Fig. 2, therefore the effects induced by quadrupole and/or hexadecapole deformations are practically indistinguishable in Fig. 2.

The values of the S factor evaluated for the spherical shape of the ^{12}C surface are slightly higher than the ones obtained for the hexadecapole deformed shape $\beta_4 = 0.16$ of ^{12}C (see Fig. 3). The strong oblate quadrupole deformation $\beta_2 = -0.4$ increases $\sim$1.7 times the $^{12}C + ^{12}C$ S-factor values at very low energies in comparison with the one obtained for the spherical shapes of colliding nuclei. The simultaneous accounting of the quadrupole and hexadecapole deformations leads to a slightly smaller enhancement of the S factor at low energies as compared with the case of pure quadrupole deformation. So the accounting of the hexadecapole deformation slightly reduces the S-factor values.

$^{12}\mathrm{C} + {}^{12}\mathrm{C}$ S-factor values evaluated with the influence of oblate quadrupole and hexadecapole deformations of the $^{12}\mathrm{C}$ shape at off-resonances energies can be parametrized by a simple formula

$$S(E) = 10^{s_0 + s_1 E + \frac{1}{2} s_2 E^2}, \tag{9}$$

where $S(E)$ is given in MeV b, $s_0 = 16.681$, $s_1 = -0.07468\ \mathrm{MeV}^{-1}$, and $s_2 = -0.04231\ \mathrm{MeV}^{-2}$. This formula describes the S-factor values with an accuracy better than 5% in the range of center-of-mass collision energy E between 2 and 5.6 MeV. However, this simple parametrization of the S factor has large errors at the higher values of E because slopes of both the S factor and the fusion cross section (see Figs. 2 and 3) are changed around the barrier of the $^{12}\mathrm{C} + {}^{12}\mathrm{C}$ interaction potential, which is close to 6 MeV. Due to the strong variation of the slope around the barrier it is impossible to find a simple and precise expression that satisfactorily describes the S-factor values at subbarrier and over-barrier energies. Noticeable variations of the fusion cross-section and/or S-factor slopes near interaction barriers are common for reactions between heavy ions, see Figs. 2, 3, and also Fig. 1 in [14,16] and Figs. 3–21 in [15]. It is also useful to note that the values of the S factor for over-barrier energies are not very important for astrophysics applications because the Gamow energy E_G for a temperature of a carbon detonation supernova 5×10^8 K is close to 1.5 MeV [1,12].

Note values of both the S factor and the fusion cross section depend on the value and the sign of the parameter of the quadrupole deformation β_2. Thus, the less-deformed oblate shape of the $^{12}\mathrm{C}$ surface leads to a smaller value of the S factor in comparison with the one presented in Fig. 3. However, strong enhancement of the fusion cross-section values at subbarrier energies can be observed in the case of the significant prolate $\beta_2 > 0$ deformation. It is useful to note here that the value of the dynamical quadrupole deformation $\beta_2 = +0.582$ is evaluated for the lowest 2^+ vibrational state 4.44 MeV in $^{12}\mathrm{C}$ [33]. Therefore, accurate determination of the value and sign of the quadrupole deformation parameter of the $^{12}\mathrm{C}$ ground-state shape is very important for careful evaluation of the $^{12}\mathrm{C} + {}^{12}\mathrm{C}$ fusion cross section deeply below the barrier.

In conclusion, the S factor and fusion cross-section values for the $^{12}\mathrm{C} + {}^{12}\mathrm{C}$ reaction evaluated with the exact account of quadrupole and hexadecapole deformations and averaging all possible mutual orientations of colliding nuclei agree well with the majority of the experimental data at off-resonances energies around barrier and well-below barrier.

The S factor and the fusion cross-section values for the $^{12}\mathrm{C} + {}^{12}\mathrm{C}$ reaction are enhanced due to the influence of the strong oblate quadrupole deformation of $^{12}\mathrm{C}$. The hexadecapole surface deformation of $^{12}\mathrm{C}$ weakly reduces both the fusion cross-section and the S-factor values.

The accounting of the surface deformation of $^{12}\mathrm{C}$ is important for the accurate evaluation of the S factor for very low collision energies. The most adequate description of the fusion cross-section values for the $^{12}\mathrm{C} + {}^{12}\mathrm{C}$ reaction at subbarrier energies should be done in the framework of this approach, which takes into account both the quadrupole and hexadecapole deformations of colliding nuclei and the contribution of molecular resonances into the total fusion cross section.

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