Collective modes and response functions of relativistic asymmetric nuclear matter

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Response functions and collective modes for asymmetric nuclear matter ($N > Z$) are studied by means of a relativistic kinetic equation. Nuclear matter is described by means of a field theory of nucleons coupled to neutral, scalar (σ), and vector (ω) mesons and to charged vector mesons (ϱ). The contribution of the exchange terms to the nucleon-nucleon interaction is also taken into account. Because of the exchange terms, the kinetic equation presents solutions which correspond to oscillations of the spin density. Finally the effects of an excess of neutrons on the response functions and on the collective modes are investigated. It is found that a collective oscillation present in symmetric nuclear matter disappears even for a small asymmetry ($\sim 5\%$).

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A relativistic approach to the study of nuclear dynamics known as quantum hadrodynamics (QHD) has been developed in Refs. [1] and [2]. Various properties of nuclear matter have already been investigated in the framework of this theory, however, in most of the previous works collective modes and response functions of symmetric (equal number of neutrons and protons, $N = Z$) nuclear matter [3–7] and of neutron matter [8] have been studied either in the simplified version of this theory known as mean-field theory (MFT) or in the one-loop approximation. In both cases only the direct terms of nucleon-nucleon interaction have been taken into account. Moreover, the collective excitations and response functions of nuclear matter have been studied only in the isoscalar channel.

Here we investigate the response functions and collective excitations of cold nuclear matter by extending a relativistic kinetic equation derived in a previous work [9] to include isospin degrees of freedom. In such a way we can study response functions and collective excitations both in the isoscalar and isovector channels. The kinetic equation of Ref. [9] has been derived in a mean-field approximation within the framework of version I of QHD (QHD-I) [1]; in the present calculations we also include the charged ϱ meson field in a phenomenological way, similar to the MFT approach to QHD-II of Ref. [1]. Our study is extended also to asymmetric nuclear matter in order to investigate possible new effects introduced by an excess of neutrons and in order to provide a unified description of nuclear matter and neutron stars.

We do take into account also the contribution of exchange terms to the mean field since, as shown in Ref. [9], these terms play a crucial role. Actually, within a theory where only scalar and vector mesons are considered, excitation of spin-density collective modes in nuclear matter is possible only if exchange terms are included. Moreover, in models without charged meson fields, the isoscalar and isovector channels are degenerate if the exchange terms are neglected.

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The ϱ meson field is treated in the same approximation introduced in Ref. [9] for the neutral scalar σ and vector ω mesonic fields: We assume that in the equations for the mesonic fields the terms containing derivatives can be neglected with respect to the mass terms (see Eqs. (2.1) of Ref. [9]). Therefore, in this approximation, the ϱ-field operator $\mathbf{B}^\mu(x)$ is directly connected to the operator of the isovector current density:

$$\mathbf{B}^\mu(x) = \frac{g_\varrho}{m_\varrho^2}\bar{\psi}(x)\gamma^\mu\boldsymbol{\tau}\psi(x), \tag{1}$$

with $\psi(x)$ the eight-component nucleon field, g_ϱ the ϱ-N coupling constant, and m_ϱ the mass of the ϱ meson.

The assumptions expressed by Eq. (1) and by the analogous equations for the neutral mesonic fields σ and ω (Eqs. (2.3) of Ref. [9]) limit the validity of our approach to the region of small momentum transfer; however, we are able to take into account several effects in a relativistically covariant formulation of nuclear dynamics.

First, we briefly examine the particle distribution in four-momentum space for the asymmetric nuclear matter at equilibrium. The proton and neutron Fermi momenta are p_{F_1} and p_{F_2}, respectively. Proceeding as in Ref. [9] we obtain

$$N_{(1)}(p) = \frac{2}{(2\pi)^3}\Theta\left(\sqrt{p_{F_1}^2 + M_{(1)}^{*2}} - \sqrt{\mathbf{p}^2 + M_{(1)}^{*2}}\right) \times\delta\left(\Pi^0 - f'_V b^0 - \sqrt{\mathbf{p}^2 + M_{(1)}^{*2}}\right), \tag{2a}$$

with $M_{(1)}^* = M^* - f'_S b$ for the proton distribution, and

$$N_{(2)}(p) = \frac{2}{(2\pi)^3}\Theta\left(\sqrt{p_{F_2}^2 + M_{(2)}^{*2}} - \sqrt{\mathbf{p}^2 + M_{(2)}^{*2}}\right) \times\delta\left(\Pi^0 + f'_V b^0 - \sqrt{\mathbf{p}^2 + M_{(2)}^{*2}}\right), \tag{2b}$$

with $M_{(2)}^* = M^* + f'_S b$ for the neutron distribution. The quantities b and b^0 are given by the following expectation values on the ground state

$$b = \langle\bar{\psi}(x)\tau_3\psi(x)\rangle \tag{3a}$$

and

$$b^0 = \langle \bar{\psi}(x)\tau_3\gamma^0\psi(x)\rangle. \tag{3b}$$

The quantities Π^0 and M^* are the isoscalar parts of the fourth component of the kinetic momentum and of the effective mass, respectively,

$$\Pi^0 = p^0 - \tilde{f}_V \rho_B, \qquad M^* = M - \tilde{f}_S \rho_S, \tag{4}$$

where ρ_S and ρ_B represent the scalar and baryon densities. The effective values of the isoscalar coupling constants $\tilde{f}_S$ and $\tilde{f}_V$ and of the isovector coupling constants f'_S and f'_V contain the contributions of exchange terms. They are given by the combinations

$$\begin{aligned}
\tilde{f}_S &= \frac{7}{8}f_S + \frac{1}{2}f_V + \frac{3}{2}f_\varrho, \\
\tilde{f}_V &= \frac{1}{8}f_S + \frac{5}{4}f_V + \frac{3}{4}f_\varrho, \\
f'_S &= -\frac{1}{8}f_S + \frac{1}{2}f_V - \frac{1}{2}f_\varrho, \\
f'_V &= \frac{1}{8}f_S + \frac{1}{4}f_V + \frac{3}{4}f_\varrho,
\end{aligned} \tag{5}$$

with $f_V = (g_V/m_V)^2$, $f_S = (g_S/m_S)^2$, and $f_\varrho = (g_\varrho/m_\varrho)^2$. We remark that in our approach both the ground state and the dynamical properties are determined only by the ratios between the meson coupling constants and the meson masses.

In our model the nucleon effective mass takes different values for protons and neutrons. If the exchange terms are neglected, the isovector coupling constant f'_S vanishes; in this case the effective masses of protons and neutrons would coincide even in asymmetric nuclear matter. Moreover, it should be remarked that exchange terms give rise to isovector contributions to the effective mass and to the energy of nucleons, even without including the charged meson field. The ϱ field has the only effect of changing the effective values of coupling constants.

In the nuclear case ($N/Z \approx 1.5$) the ratio $M^*_{(1)}/M^*_{(2)}$ differs from unity only by a few percent, but for a fixed asymmetry, this ratio increases rapidly with the baryon density. For densities and values of the asymmetry typical of neutron stars the difference between the effective masses of protons and neutrons cannot be neglected, since in this case the value of $M^*_{(1)}/M^*_{(2)}$ can be as high as ≈ 2.5–3.

We choose the value of the coupling constants f_V and f_S in order to reproduce the binding energy (15.75 MeV) of saturated symmetric nuclear matter with a Fermi momentum of 1.3 fm^{-1}. These values are

$$f_S = 3.18 \times 10^{-4}\ \text{MeV}^{-2}, \quad f_V = 2.074 \times 10^{-4}\ \text{MeV}^{-2}.$$

For the coupling constant f_ϱ we have taken the value determined by the $\varrho \to 2\pi$ decay

$$f_\varrho = 1.55 \times 10^{-5}\ \text{MeV}^{-2}.$$

With these values the symmetry energy and the bulk modulus turn out to be $a_4 = 42$ MeV and $K_V^{-1} = 540$ MeV. This value of K_V^{-1} is substantially higher than the commonly accepted experimental value of 200–350 MeV [10]. However, it is very close to those obtained in the Hartree or Hartree-Fock approximations of QHD without nonlinear terms of the scalar field in the original Lagrangian [11]. Our value of a_4 instead is somewhat

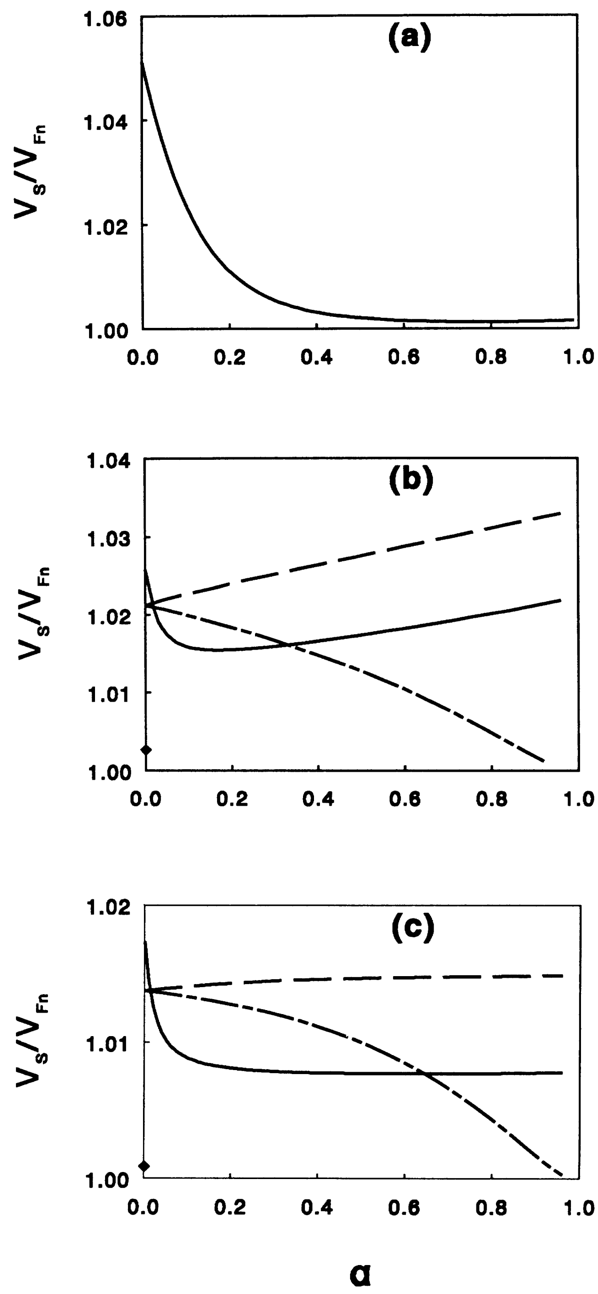

FIG. 1. Ratio of zero-sound velocities to the neutron Fermi velocity v_{F_n} as a function of the asymmetry parameter α. (a) Longitudinal current-density waves. (b) Longitudinal spin-density waves. The solid line corresponds to isovector waves. For isoscalar waves the result is shown only for $\alpha = 0$ (♦). The dashed line and the dot-dashed line correspond to the velocities of neutron and proton waves, respectively, calculated without the ϱ field. (c) The same as in (b) but for transverse spin-density waves.

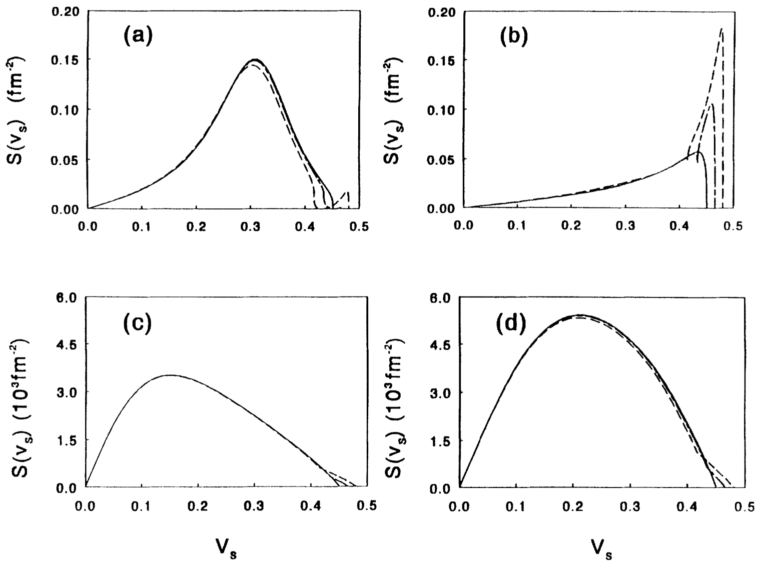

FIG. 2. Vector response functions of asymmetric nuclear matter for three different values of the asymmetry parameter: $\alpha = 0.0$ (solid line), $\alpha = 0.1$ (dot-dashed line), and $\alpha = 0.2$ (dashed line). (a) Longitudinal isoscalar-isoscalar component. (b) Longitudinal isovector-isovector component. (c), (d) The same as (a) and (b), respectively, but for the transverse components.

larger than that given by the Hartree-Fock approximation, $a_4 = 35$ MeV [11]. This is because our approximation overestimates exchange contributions. However, we notice that the value of a_4 results from the mutual subtraction of two very large terms, so that a relatively small overestimate of each term can influence the value of the symmetry energy appreciably.

Here we calculate the linear response functions of asymmetric nuclear matter in the various spin-isospin channels; therefore we add small external fields to the nuclear mean field in the kinetic equation of Ref. [9]. Moreover, we consider isovector density fluctuations with $m_T = 0$ only; i.e., we do not study processes where a neutron converts into a proton or vice versa. In a linear approximation, oscillations with $m_T = \pm 1$ are decoupled from those with $m_T = 0$.

Our approach is based on a semiclassical description of nuclear matter dynamics, so that the nucleon densities (scalar, vector, tensor, pseudovector, and pseudoscalar) are assumed to be smooth functions of the space-time coordinates. Within this approximation the equations for the scalar and current densities are decoupled from those for the pseudovector and tensor densities (see Ref. [9]). Moreover, from the structure of the resulting equations one can see that the oscillations in the direction of the wave vector **k** are decoupled from transverse oscillations.

The first object of our investigation is the behavior of the collective modes of nuclear matter as a function of the asymmetry $\alpha \equiv (N - Z)/(N + Z)$. We study both current- and spin-density oscillations. The current-density and the density oscillations are related by the continuity equations for the electric and baryon currents. The collective modes are characterized by their propagation velocity. As shown in Ref. [9], the collective solutions do not depend on k^0 and $|\mathbf{k}|$ separately, but only on the velocity $v_s = |\mathbf{k}|/k^0$. This implies that these modes are of the zero-sound type. In Fig. 1 the ratio of v_s to the neutron Fermi velocity v_{F_n} is plotted for the relevant channels.

We note that current-density oscillations propagate only through longitudinal waves, while in spin-density oscillations both longitudinal and transverse waves are possible. For symmetric nuclear matter the current-density oscillations correspond to oscillations of the isovector

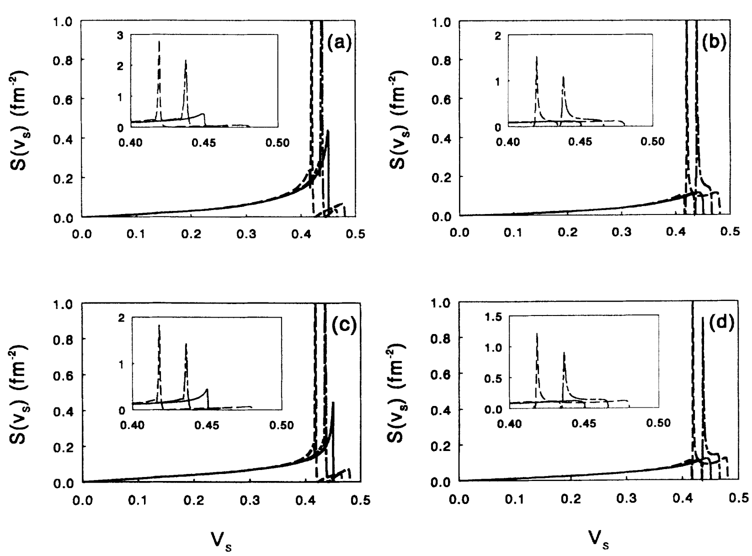

FIG. 3. The same as Fig. 2, but for axial response functions.

density with $m_T = 0$; i.e., neutrons and protons oscillate out of phase. For these waves an interesting feature emerges from our relativistic calculations: the collective solution survives for all values of α [see Fig. 1(a)]. This is in contrast with the results of previous treatments of the same subject based on the nonrelativistic Landau theory of Fermi liquids [12,13]. The Landau theory predicts that density oscillations become strongly damped beyond a critical value of α (in Ref. [12] this has been shown to occur for $\alpha > 0.6$). This difference between our results and those based on the Landau theory can be ascribed to a relativistic effect.

As a general remark we notice that the neutron excess plays a qualitatively similar role in all channels investigated: with increasing asymmetry the repulsive part of the nucleon-nucleon interaction becomes softer and softer. In the isoscalar case of the spin-density oscillations this is sufficient to make the collective mode become a damped oscillation for a very small value of α ($\alpha \simeq 0.05$). The role played by the ϱ meson, instead, depends critically on the channel considered. For current-density oscillations the ϱ meson does not affect the results appreciably, while in spin-density oscillations the ϱ meson changes qualitatively the behavior of results. This can be appreciated from Figs. 1(b) and 1(c), where the zero-sound velocities calculated with and without the ϱ meson are reported. Without the ϱ meson the proton and neutron spin-densities can oscillate independently; when the ϱ meson is included there is a coupling between the two densities because nucleons can now exchange a charged particle. The difference between the current- and spin-density oscillations is due to the fact that for spin density only the exchange terms contribute to the self-consistent mean field.

We turn now to the response functions. They are defined by

$$S(k) = -\frac{1}{\pi}\,\mathrm{Im}\Pi(k), \tag{6}$$

where $\Pi(k)$ represents the polarization propagator in the relevant particle-hole p-h channel.

In Figs. 2 and 3 we present four vector response functions and four axial response functions, for three different values of the asymmetry parameter α, which may be of interest in the nuclear case. The collective modes have not been shown in the figures; their positions and relative strengths, for the channels in which they occur, are summarized in Table I. In our semiclassical approximation the response functions display a sort of scaling property in the sense that they do not depend on k^0 and $|\mathbf{k}|$ separately, but only on the ratio $v_s = k^0/|\mathbf{k}|$.

TABLE I. Position (v_s/v_{F_n}) and the relative strength (w) of collective modes for the various channels. Channel 1 corresponds to isovector longitudinal oscillations of the current density. Channels 2 and 3 correspond to longitudinal oscillations of the spin density: isoscalar oscillations in channel 2 and isovector oscillations in channel 3. Channels 4 and 5 are the same as channels 2 and 3, respectively, but for transverse oscillations.

	$\alpha = 0.0$		$\alpha = 0.1$		$\alpha = 0.2$	
Channel	v_s/v_{F_n}	w	v_s/v_{F_n}	w	v_s/v_{F_n}	w
1	1.051	0.67	1.024	0.61	1.011	0.52
2	1.002	0.05				
3	1.026	0.24	1.016	0.13	1.015	0.11
4	1.001	0.02				
5	1.018	0.16	1.009	0.07	1.008	0.06

In the channels where collective modes occur, the response functions show peaks that are more and more pronounced with increasing neutron excess. This is because the position of the undamped modes approaches the upper bound of the continuum of the p-h excitations as the asymmetry increases. The neutron excess gives rise to another effect that can be understood in a rather straightforward way. It consists in the shift of the upper bound of the p-h continuum ($v_s/v_{F_n} = 1$) towards higher values of v_s. This is simply due to the increase of the neutron Fermi velocity with increasing asymmetry. As a general remark we observe that in all the channels the total strength of the response function does not change appreciably within the considered values of α. The effect of the asymmetry consists essentially in redistributing the strength of the response functions between collective modes and the continuum of the p-h excitations with different weights.

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